MATH 234 — QUIZ 05 — OCTOBER 2014

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Problem 1 (5 points): Find the equation for the plane tangent to the graph of $f(x, y) = x\ln(xy)$ at $(2, 1/2, 0)$.

Solution:

$$f_x(x, y) = \ln(xy) + x\frac{1}{xy}y \implies f_x(2, \frac{1}{2}) = 1$$
$$f_y(x, y) = x\frac{1}{xy}x \implies f_y(2, \frac{1}{2}) = 4$$

So the tangent plane to the graph at $(2, 1/2, 0)$ is given by $z = 0 + 1 \cdot (x - 2) + 4 \cdot (y - \frac{1}{2}) \iff z = x + 4y - 4$.

Problem 2 (5 points): Given the coordinate transformation $x = \cos(\theta)u - \sin(\theta)v, y = \sin(\theta)u + \cos(\theta)v$

1. If $\theta = \pi/4$, what are the coordinate (x, y) of the point $(u, v) = (1, 0)$.
2. Use x, y and θ to represent u, v, i.e. write down the inverse coordinate transformation.
3. Use the coordinate x, y to represent the zero set $f(u, v) = u^2 - v^2 - 1 = 0$.

Solution:

1. $\sin(\pi/4) = \sqrt{2}/2, \cos(\pi/4) = \sqrt{2}/2$. So

$$x = \cos(\theta)u - \sin(\theta)v = \frac{\sqrt{2}}{2} \cdot 1 - \frac{\sqrt{2}}{2} \cdot 0 = \frac{\sqrt{2}}{2}$$
$$y = \sin(\theta)u + \cos(\theta)v = \frac{\sqrt{2}}{2} \cdot 1 + \frac{\sqrt{2}}{2} \cdot 0 = \frac{\sqrt{2}}{2}$$

2.

$$\begin{aligned} u &= \cos(\theta)x + \sin(\theta)y \\ v &= -\sin(\theta)x + \cos(\theta)y \end{aligned}$$

3.

$$\begin{aligned} F(x, y) &= f(u(x, y), v(x, y)) \\ &= (\cos(\theta)x + \sin(\theta)y)^2 - (\cos(\theta)y - \sin(\theta)x)^2 - 1. \end{aligned}$$