

Surface Integrals

1. Find a parametrization of the following surfaces:

(a) The cone: $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.

(b) The sphere $x^2 + y^2 + z^2 = a^2$

(c) The cylinder $x^2 + (y - 3)^2 = 9$, $0 \leq z \leq 5$.

(d) The cap cut from the sphere $x^2 + y^2 + z^2 = 9$ by the cone $z = \sqrt{x^2 + y^2}$.

2. Find the surface area of the cone in 1 (a)

3. Let S be the "football" surface formed by rotating the curve $x = \cos z, y = 0, -\pi/2 \leq z \leq \pi/2$ around the z -axis. Find a parametrization for S and compute its surface area.

Solutions:

1. (a) $\vec{x}(\theta, z) = \begin{pmatrix} z \cos \theta \\ z \sin \theta \\ z \end{pmatrix}, 0 \leq \theta < 2\pi, 0 \leq z \leq 1.$

(b) $\vec{x}(\theta, \phi) = \begin{pmatrix} a \sin \phi \cos \theta \\ a \sin \phi \sin \theta \\ a \cos \phi \end{pmatrix}, 0 \leq \theta < 2\pi, 0 \leq \phi \leq \pi.$

(c) $\vec{x}(\theta, z) = \begin{pmatrix} 3 \cos \theta \\ 3 + 3 \sin \theta \\ z \end{pmatrix}, 0 \leq \theta < 2\pi, 0 \leq z \leq 5.$

(d) $\vec{x}(\theta, \phi) = \begin{pmatrix} 3 \sin \phi \cos \theta \\ 3 \sin \phi \sin \theta \\ 3 \cos \phi \end{pmatrix}, 0 \leq \theta < 2\pi, 0 \leq \phi \leq \frac{\pi}{4}.$

2. $\sqrt{2}\pi$

3. $2\pi[\sqrt{2} + \ln(1 + \sqrt{2})]$