

Triple Integrals

1. Evaluate the following integrals:

(a) $\int_0^\pi \int_0^\pi \int_0^\pi \cos(x + y + z) dx dy dz$

(b) $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz dy dx$

(c) $\int_0^1 \int_0^{1-x^2} \int_3^{4-x^2-y} x dz dy dx$

2. Rewrite the integral

$$\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} dz dy dx$$

as an equivalent iterated integral in the order:

- (a) $dydzdx$
- (b) $dydxdz$
- (c) $dx dy dz$
- (d) $dx dz dy$
- (e) $dz dx dy$

3. Find the volume of the region cut from the solid elliptical cylinder $x^2 + 4y^2 \leq 4$ by the xy -plane and the plane $z = x + 2$.

4. Find the average value of $F(x, y, z) = xyz$ over cube in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$.

Solutions:

1. (a) 0

(b) 18

(c) $\frac{1}{12}$

2. (a) $\int_{-1}^1 \int_0^{1-x^2} \int_{x^2}^{1-y} dydzdx$

(b) $\int_0^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{x^2}^{1-z} dydxdz$

(c) $\int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} dx dy dz$

(d) $\int_0^1 \int_0^{1-y} \int_{-\sqrt{y}}^{\sqrt{y}} dx dz dy$

(e) $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{1-y} dz dx dy$

3. 4π

4. 1