Math 217 2011 Spring
Worksheet 6

Name: ________________________________

1. Find the integral
a) \( \int \frac{5y}{\sqrt{2y^2 - 3}} \, dy \)
b) \( \int_{-2}^{2} \sqrt{4 - x^2} \, dx \)
c) \( \int \cosh \sqrt{y} \, dy \)
d) What is the average value of \( f(x) = \sqrt{5 - x} \) on the interval \([0, 5]\)?
e) \( \int \sqrt{1 - x^4} \, dx \)
f) \( \int_{0}^{\frac{\pi}{2}} \sin 3x \, dx \)
g) \( \int_{\ln(\ln 3)}^{\ln(\ln 2)} e^x e^x \, dx \)
i) \( \int e^{-x} e^x \, dx \)
j) \( \int_{0}^{1} \frac{x - 1}{\sqrt{x^2 + x}} \, dx \)
k) \( \int \sin^3 3t \sqrt{\cos 3t} \, dt \)
l) \( \int 2e^x \sinh x \, dx \)

2. a) At 10:00 am one care was 150 miles due west from a second car. If the first car drove east at 50 miles per hour and the second car drove southwest at 75 miles per hour, when were they closest together?
b) Plot the graph of \( f(x) = \frac{3 - x^2}{x - 2}, \, x \neq 2 \). Include the coordinates of any local extreme points and any inflection points.

3. Find \( \frac{dy}{dx} \) for
a) \( y = \sqrt{e^{x^2}} + e^{\sqrt{x^2}} \)
b) \( y = \int_{-1}^{x} \cos^2 t \, dt \)
c) \( y = \frac{\sqrt{x + 5} (2x + 1)}{(x - 1) \sqrt{3x - 4}} \)
d) \( 2^x y = 2 + x \tan^{-1} y \)
e) \( y = \cosh(x^2 + 2x + 5) + 2^x + x^x \)
f) \( y = \sin x + (\cos x)^x \)
g) \( y = \cosh^{-1}(\sinh x) \)

h) \( f(x) = x^{\sin x} \). Find \( f'(\frac{\pi}{2}) \).

i) \( y = \ln\left(\frac{e^x}{1 + e^x}\right) + \log_3 e^x \)

4. a) Compute the sum \( \sum_{i=100}^{200} \left[ \frac{1}{i^2} - \frac{1}{(i+1)^2} \right] \)

b) Compute the sum \( \sum_{k=3}^{50} (k - 1)(k - 2) \)

c) Compute \( \int_{-2}^{1} (2x^2 - 3)dx \) using the definition of the definite integral (Riemann Sum)

5. a) Find the length of the curve given parametrically by \( x = e^t \sin t, \ y = e^t \cos t \), \( 0 \leq t \leq 2\pi \).

b) Find the general solution of the differential equation \( \frac{dy}{dx} = \sqrt{x} y \); then find the particular solution that satisfies the condition \( y = 4 \) at \( x = 1 \).

c) Let \( f(x) = x^3 - 3x^2 - 1, \ x \geq 2 \). Find the value of \( \frac{df^{-1}}{dx} \) at the point \( x = -1 = f(3) \).

6. a) If the brakes of a car, when fully applied, produce a constant deceleration of 9 feet per second, what is the shortest distance in which the car can be braked to a half from a speed of 60 miles per hour? (60 miles per hour is the same as 88 feet per second.)

b) You are planning to make an open box from a 2 by 2 ft. piece of cardboard, by cutting four congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make in this way, and what is its volume?

7. a) A bacterial population grows at a rate proportional to its size. Initially, it is 20000, and after 5 days it is 70000. What is the population after 9 days?

b) How long does it take for money to double at 6% interest compounded monthly?

c) The 1985 population estimate for India was 762 million, and the population has been growing continuously at a rate of about 2.2% per year. Assuming that this rapid growth rate continues, estimate the population \( N(t) \) of India in the year 2010.

d) Suppose the amount of oil pumped from an oil well in Kalamazoo, Michigan, decreases at a continuous rate of 14% per year. When will the well’s output fall to one-fourth of its present value?
8. a) Sketch the region bounded by \( x = -6y^2 + 4y \) and \( x + 3y - 2 = 0 \), and find the area of this region.
b) Sketch the region bounded by \( y = \sinh x \), \( y = 0 \) and \( x = \ln 2 \), and calculate the area of this region.
c) The base of a solid is the first quadrant plane region bounded by \( y = 2 - \frac{x^2}{2} \), the x-axis and the y-axis. The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.
d) Sketch the region R bounded by the curves \( y = \ln x \), \( y = 1 \) and \( x = 1 \). Set up, but do not evaluate the integral which gives the area of R. Set up, but do not evaluate the integral which gives the volume of the solid generated by rotating R about the x-axis. Set up, but do not evaluate the integral which gives the volume of the solid generated by rotating R about the y-axis.

9. Compute the limits
   a) \( \lim_{x \to \frac{\pi}{2}} \frac{\ln(\sin x)^2}{x - \frac{\pi}{2}} \)
   b) \( \lim_{x \to 0} \frac{x - \tan^{-1} x}{2x^3} \)
   c) \( \lim_{\theta \to \frac{\pi}{2}} (\sec \theta - \tan \theta) \)
   d) \( \lim_{x \to 0^-} (\cos x)^{\frac{5}{x}} \)
   e) \( \lim_{x \to +\infty} \sum_{i=1}^{n} \frac{(2i - 1) i - 1}{n} \)
   f) \( \lim_{x \to 0} \frac{\sin x - \tan x}{x^2 \sin x} \)
   g) \( \lim_{x \to -\infty} (\tan^{-1} x - \frac{1}{x}) \)
   h) \( \lim_{x \to 0^+} (x^2)^x \)

10. a) Write the expression \( \frac{4 + \sqrt{-81}}{i - \sqrt{-64}} \) in the form \( a + bi \), where \( a \) and \( b \) are real numbers.
b) Find all real and complex solutions of the equation \( x^3 + 125 = 0 \)
c) Write the complex number \( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \) in polar form, then evaluate \( \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)^{2010} \)
d) Suppose that \( w \) and \( z \) are complex numbers. Show that \( |w + z| \leq |w| + |z| \).

11. Evaluate the following expressions
   a) \( \sin^{-1}(\sin \frac{5\pi}{4}) \)
   b) \( \tan(\cos^{-1} x) \)
12. Solve for x
a) $e^x + 2 = 8e^{-x}$
b) $\log_2 x - \log_2 (x + 1) = \log_2 \left(\frac{1}{8}\right)$
c) $\frac{1}{2} \sin^{-1}(x - 3) = \frac{\pi}{4}$