1. We have three biased coins: the probability of heads for the first one is 3/4, for the second one is 3/8 and for the third one is 1/4. We also have a fair die. We roll the die and flip one of the three coins: if the number on the die is 1 or 2 then we use the first coin, if it’s 3 or 4 then we use the second, otherwise the third. Compute the conditional probability that we used the first coin given that the coin landed on tails.

**Solution.** Let $B_i$ denote the event that we chose the $i^{th}$ coin and $A$ that the coin landed on tails. Then

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}, \quad P(A|B_1) = \frac{1}{4}, \quad P(A|B_2) = \frac{5}{8}, \quad P(A|B_3) = \frac{3}{4}.$$

We need to compute $P(B_1|A)$ and by the Bayes’ formula

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)} = \frac{\frac{1}{3} \cdot \frac{1}{4}}{\frac{1}{3}(\frac{1}{4} + \frac{5}{8} + \frac{3}{4})} = \frac{2}{13}.$$ 

2. We roll two dice. Let $A$ be the event that the first die shows 1 and $B$ the event that the sum is equal to 7. Show that the two events are independent.

**Solution.** We have $P(A) = \frac{1}{6}$ and $P(B) = \frac{1}{6}$ (since there are six ordered pairs out of the 36 that will have a sum equal to 7). Note that $AB$ is the event that the first die shows 1 and the second shows 6. The probability of this is $\frac{1}{36}$ which is equal to $P(A)P(B)$. This shows the independence.