1. We wrote integer numbers on the vertices of a tetrahedron (one for each vertex). Show that if the sum of the numbers on each face is divisible by five, then all the numbers are divisible by five.

2. Let $ABCD$ be a quadrilateral such that $\angle DAB + \angle ABC = 120^\circ$. Construct equilateral triangles $ACE, BDF, CDG$, with $E, F, G$ each on the other side of $AC, BD, CD$ (respectively) from $AB$. Show that $E, F, G$ are collinear (i.e. there is a line that goes through all three).

3. Let $n > 1$ be an integer and suppose that the polynomial $p(x)$ has degree $n$ and satisfies $p(1) = 3, p(2) = 5, p(3) = 7, \ldots, p(n) = 2n + 1$, and $p(n + 1) = 2n + 5$. Evaluate $p(n + 3)$.

4. Is there a function $f(x)$ defined on all real numbers $x$ so that $f(f(x)) = -x$ for all $x$? (Either prove that there is no such $f$, or give an example of one.)

5. 2013 students each roll 9 standard six-sided dice and record how many times each of the numbers 1, 2, 3, 4, 5, 6 appear. Show that there are at least two students who recorded the same result.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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