Math 222 — Final Exam, fall 97.

IMPORTANT: You do not have to do all the problems, but you must do problems 6 and 7. You must also choose three (i.e. 3) problems from 1–5. Read the problems now, and circle the numbers of the problems you will do in the following box.

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1. Let \( I_n = \int (\tan x)^n \, dx \).
   
   (a) Show that for any \( n \geq 2 \) one has \( I_n = \frac{(\tan x)^{n-1}}{n-1} - I_{n-2} \).
   
   (b) Compute \( \int \{ (\tan x)^3 + (\tan x)^2 \} \, dx \).

2. Let \( f(x) = \frac{x^3}{x^2 - 3x + 2} \).
   
   (a) Find the partial fraction expansion of \( f(x) \) (including the values of the constants.)
   
   (b) Compute \( \int f(x) \, dx \).

3. Solve the following two differential equations:
   
   (a) \( \frac{dy}{dx} = \frac{xe^{-x^2}}{\ln y} \).
   
   (b) \( \frac{dz}{dx} - \tan(x) \, z = \left( \frac{1}{\cos(x)} \right)^3 \).
   
   (c) Find the solution of equation (b) with \( z(0) = 1 \).

4. Find the radius of convergence of the following two power series.
   
   (a) \( S_1(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \)
   
   (b) \( S_2(x) = \sum_{n=0}^{\infty} \frac{x^n}{2^{2n} + 3^n} \)
5. (a) Find all terms in the Taylor Maclaurin series of \( f(x) = \frac{1}{1 + x^3} \).

(b) Compute the first five terms (i.e. those up to and including \( x^4 \)) of the Taylor-Maclaurin expansion of \( f(x) = \sin x \cos x \).

6. Let \( ABCD \) be a quadrilateral in which \( AB \) and \( CD \) are parallel.
Let \( P \) be the midpoint of \( AB \).
Let \( Q \) be the midpoint of \( CD \).
Denote the position vectors of \( A, B, C, D \) by \( a, b, c, d \), respectively.

(a) Find the position vectors \( p, q \) of \( P \) and \( Q \) in terms of \( a, b, c, d \).

(b) Show that \( PQ \) is parallel to \( AB \).

7. Let \( V \) be the plane with equation \( 2x + y + 2z = 2 \), and let \( W \) be the plane with equation \( x + 2y + 2z = 2 \). Let \( P \) be the point with coordinates \( \left( \frac{2}{2}, 0, 2 \right) \).
Let the line \( \ell \) be the intersection of the planes \( V \) and \( W \).

(a) Make a page-size drawing of the \( x, y, z \) axes with the planes \( V, W \) and the point \( P \).

(b) Find the equation of the plane which contains the point \( P \) and the line \( \ell \).

(c) What is the angle between the planes \( V \) and \( W \)? (Note: the angle between two planes is the same as the angle between their normal vectors; it's OK if you only find the Sine or Cosine of this angle.)

(d) Find a vector representation of the line \( m \) which is perpendicular to the plane \( V \), and which goes through the point \( P \).