1. Compute the following integrals

\[ A = \int (\sin x)^2 (\cos x)^4 \, dx \quad \text{and} \quad B = \int \frac{dx}{\sqrt{(5 + 4x + x^2)}} \]

2. (a) Does the integral \( \int_0^\infty \frac{x \, dx}{(x + 1)(x^2 + 2)} \) converge?

(b) Compute the integral \( \int_0^\infty \frac{x \, dx}{1 + x^4} \).

3. (a) Write the partial fraction expansion of \( \frac{x}{x^3 - 1} \). You do not have to evaluate the constants.

(b) Find the coefficient of \( \frac{1}{x - 2} \) in the partial fraction expansion of

\[ \frac{x^3 - 2x}{(x - 2)(x^2 + 2)^2} \]

4. Dr. Liebowitz’ model for the growth of the XYZ bacterium in his swimming pool states that the relative growth rate of the population is \( \frac{dy}{dt} = \frac{2,000}{10^6 + y^2} \), when the population consists of \( y \) individuals. Thus the population satisfies

\[ \frac{dy}{dt} = \frac{2,000y}{10^6 + y^2} \]

If the initial population is \( y(0) = 100 \), then when will the population reach one million?

5. In the following circuit the input and output voltages satisfy

\[ C \frac{dV_{\text{out}}}{dt} = \frac{V_{\text{in}}(t) - V_{\text{out}}(t)}{R} \]

where \( R \) is the resistance, \( C \) is the capacitance. Assuming that \( R = C = 1 \), and assuming that the input voltage is given by \( V_{\text{in}}(t) = \sin(\Omega t) \) for some constant \( \Omega \), compute the output voltage at time \( t \).