Problem 1
Consider the function
\[ f(x) = \begin{cases} \sqrt{x} \ln x & \text{when } x > 0, \\ 0 & \text{for } x = 0. \end{cases} \]

(a) What would you have to do to check if \( f \) is continuous at \( x = 0 \)?

Is \( f \) continuous at \( x = 0 \)?

(b) Find the maxima and minima of \( f \). Are they absolute?

(c) Does \( f'(0) \) exist? Find \( \lim_{x \to 0} f'(x) \).

(d) Find the inflection points of the graph of \( y = f(x) \).

(e) Does the graph of \( y = f(x) \) have a horizontal asymptote?

Problem 2
Let \( a \) be a positive number.
Consider the function \( f(x) = 2x - \ln(ax) \), which is defined for all \( x > 0 \).
It is given that the graph of \( y = f(x) \) intersects the \( x \)-axis exactly once. Find \( a \).

Problem 3
Compute the following:

(a) \[ \int_0^\pi \frac{\sin x}{1 + \cos^2 x} \, dx \]

(b) \[ \frac{d}{dx} \left( 1 + x^2 \right)^{\arctan x} \]

(c) \[ \lim_{x \to 0} \frac{e^{2x} - 2e^x + 1}{\cos x - 1} \]

Problem 4
Let \( R \) be the region of the plane contained between the graphs of \( y = \sqrt{x} \) and \( y = x^4 \), and between the lines \( x = 0 \) and \( x = 1 \).

(a) Calculate the volume of the solid obtained by revolving the region \( R \) around the \( x \)-axis.

(b) What is the volume of the solid obtained by revolving the region \( R \) around the \( y \)-axis.