Second Midterm Exam

Name:

Begin each problem on a new page. Number your pages. At the end of the exam staple all pages in order together, with this page as cover. All answers should be explained.

Scrap paper is available in large quantities.

(1) Compute the following limits

\[ A = \lim_{x \to 3} \frac{x^2 - 5x + 6}{x^2 - 9}, \quad B = \lim_{x \to 0} \frac{\cos x - 1}{x \sin x}, \]

\[ C = \lim_{x \to -1} \frac{x + 1}{\sqrt{x^2 + 2} - 1}, \quad D = \lim_{x \to 0} \sqrt{|x| \sin(1/x)}. \]

(2) Find the following derivatives

\[ f'(x), \text{ where } f(x) = \frac{x}{1 + x^3}, \]

\[ g'(x), \text{ where } g(x) = \sin(\sin x), \]

\[ h'(x), \text{ where } h(x) = \sqrt{1 + x^2}. \]

(3) True or False? In each case prove the statement, or give a counterexample.

(a) If \( \lim_{x \to a} f(x) \) does not exist and \( \lim_{x \to a} f(x) + g(x) \) does not exist, then \( \lim_{x \to a} g(x) \) does not exist.

(b) If \( \lim_{x \to a} f(x) \) does not exist but \( \lim_{x \to a} f(x) + g(x) \) does exist, then \( \lim_{x \to a} g(x) \) does not exist.

(c) There is a number \( x \in (-1, 1) \) such that \( 2x + \cos x = 0 \).

(d) If \( f : (0, 1] \to \mathbb{R} \) is any continuous function, then there always is some \( c \in (0, 1] \) at which \( f \) attains its maximal value, i.e. such that \( f(x) \leq f(c) \) for all \( x \in (0, 1] \).

(e) If a function \( f : [0, 1) \to \mathbb{R} \) is continuous, then \( f \) must be bounded.

(4) Suppose that \( f : [a, b] \to \mathbb{R} \) is a function, and \( c \in [a, b] \) is a number such that \( f(x) < 1 \) for all \( x \in [a, b] \) with \( x \neq c \). Suppose also that \( \lim_{x \to c} f(x) \) exists. Show that \( \lim_{x \to c} f(x) \leq 1 \).

(5) State and prove the Product Rule for Differentiation.