1. Determine if the following expressions make sense. If they do, compute the result. Otherwise, explain why they do not.

(a) \( \frac{1}{2} + \left( -\frac{1}{4} \right) \)
(b) \( \frac{3\pi}{8} \cdot \left( \frac{4}{x} \right) \)
(c) \( \frac{4}{5} - \left( -\frac{1}{3} \right) \cdot \left( 6 \right) \)
(d) \( \frac{d}{dt} \left[ \begin{pmatrix} 3t \\ e^t \end{pmatrix} \right] \)
(e) \( \begin{pmatrix} 3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \end{pmatrix} \)
(f) \( \frac{\partial}{\partial z} \left[ 3x^2y - 4yt \ln(s) \right] \)
(g) \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \)
(h) \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} \)
(i) \( \begin{pmatrix} 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \end{pmatrix} \)
(j) \( \begin{pmatrix} 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \end{pmatrix} \)
(k) \( \begin{pmatrix} 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \end{pmatrix} \)
(l) \( \begin{pmatrix} 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \end{pmatrix} \)

2. Find the equation of the plane through the point \((0, 0, 1)\) with normal vector \(\vec{n}\) where:

(a) \(\vec{n} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\)
(b) \(\vec{n}\) is orthogonal to \(\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}\) and \(\begin{pmatrix} 4 \\ 0 \end{pmatrix}\)
(c) \(\vec{n}\) is parallel to the gradient of \(f(x, y, z) = x + y^2 + z^3\) at \((0, 0, 1)\)
(d) \(\vec{n}\) is the answer to question 1,(j) above.

3. For each of the specified functions \(f(x, y)\) and points \(p\) do the following:

- Convert the function to a function \(g(r, \theta)\) in polar coordinates
- Find the partial derivatives of \(f\) with respect to \(x, y, r, \theta\).
- Find \(\frac{\partial^2 f}{\partial x \partial y}\)
- Find the tangent plane to the surface \(z = f(x, y)\) at the point \(p\).
- Find and draw the level sets of \(z = f(x, y)\) at \(z = 0, 2, -1\).
- Suppose \(x(t) = t + \ln(t)\) and \(y(t) = \cos(t)\), and find \(\frac{df}{dt}\).
(a) \( f(x, y) = x^2 + y^2 \), \( p = (3, 2, 13) \)  
(b) \( f(x, y) = xy \), \( p = (1, 1, 1) \)  
(c) \( f(x, y) = \sin(x) \), \( p = (0, 0, 0) \)  
(d) \( f(x, y) = e^x y \), \( p = (1, 1, e) \)

4. For each of the specified curves \( \vec{x}(t) \) and numbers \( u \) do the following:
   - Find \( \vec{x}'(t) \)
   - Find \( \vec{x}''(t) \)
   - Find the arc length of \( \vec{x}(t) \) from \( t = 0 \) to \( 1 \)
   - Find the unit tangent vector to \( \vec{x}(t) \) at the point \( \vec{x}(u) \)
   - Find \( \frac{df}{ds} \) the arc length derivative with respect to \( \vec{x}(t) \) for \( f(t) = \ln(t) \)
   - Find the tangent line to \( \vec{x}(t) \) at \( t = u \)
   - Find the tangent line to \( \vec{x}(t) \) for arbitrary \( t \)
   - Find the curvature vector of \( \vec{x}(t) \)
   - Find the Binormal vector to \( \vec{x}(t) \)
   - Give an equation for the osculating plane at \( t \) in the standard form \( A(t)x + B(t)y + C(t)z = D(t) \). (Hint: The Binormal vector is normal to the osculating plane)

   (a) \( \vec{x}(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ t \end{pmatrix} \)  
   (b) \( \vec{x}(t) = \begin{pmatrix} 1 \\ t^2 - 1 \\ t^2 \end{pmatrix} \)  
   (c) \( \vec{x}(t) = \frac{1}{6} \begin{pmatrix} t^6 \\ 3t^4 \\ 6t^2 \end{pmatrix} \)

5. Suppose an airplane takes off, and the curve it traces in the air has the form

   \[ \vec{x}(t) = \begin{pmatrix} t \\ 0 \\ 1 - e^{-t} \end{pmatrix} \]

   For \( t > 0 \). Assuming that the nose of the airplane is always facing the direction the plane is flying, and that the airplane rests in the osculating plane of the curve at all times, what direction are the wings of the airplane pointing in? (Hint: Your answer should be in the form \( \pm \vec{a} \) for some vector \( \vec{a} \), because there are two wings pointing in the opposite direction)

6. What is the linear approximation for the function \( f(x, y) = e^{x+y} + \ln(2x + 1) \) at the point \( (0, 0, 1) \)? Use this to approximate the numbers \( e \) and \( \ln(2) \). (HINT: \( e = f(0, 1) \) and \( \ln(2) = f(1/2, -1/2) \))