Math 641, Fall 1999
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Exercise Set 4, * exercises due Friday, October 29, 1999

Let $C$ be an $[n, k, d]$ code. The support of $C$ is the number of coordinates where not all codewords of $C$ equal zero (it is the number of nonzero columns of a generator matrix of $C$). For $1 \leq r \leq k$ the $r$th generalized Hamming weight of $C$ is the minimum support of a $r$-dimensional subcode of $C$. So $d_1$ is the minimum support of a 1-dimensional subcode, i.e. the minimum support of a non-zero vector in $C$, i.e. the minimum weight of a nonzero codeword. The numbers $d_1, d_2, \ldots, d_k$ are called the weight hierarchy of $C$.

* 1. Prove

$$d = d_1 < d_2 < \cdots < d_k \leq n.$$  

* 2. Prove the generalized Singleton bound

$$d_r \leq n - k + r \quad (1 \leq r \leq k),$$

and show that MDS codes meet this bound for all $r$.

* 3. Determine the weight hierarchy of the $[15, 4, 8]$ binary simplex code and the Reed-Muller code $RM(1, 4)$