Math 240, Spring Semester 2000-01
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Exam 1: March 7, 2001

1. Consider the two integers 177 and 141.
   (a) Determine their GCD (show your work):

   (b) Write the GCD as a linear combination of 177 and 141
       (GCD = $a \cdot 177 + b \cdot 141$ with $a$ and $b$ integers)(show your work):

   (c) The LCM of 177 and 141 (show your work)?

2. Determine the truth value (True, False, or Need More Information) of the following statements:
   (a) If $-2$ is a positive number, then $\sqrt{-1}$ is a real number.
       
       \begin{tabular}{ccc}
       TRUE & FALSE & CAN'T SAY \\
       \end{tabular}

   (b) $\exists y \forall x \ 2x - 3y = 5$ ($x$ and $y$ designate real numbers)
       
       \begin{tabular}{ccc}
       TRUE & FALSE & CAN'T SAY \\
       \end{tabular}

   (f) $\neg(\exists x P(x)) \Rightarrow \forall x (\neg P(x))$
       
       \begin{tabular}{ccc}
       TRUE & FALSE & CAN'T SAY \\
       \end{tabular}
3. Prove using mathematical induction:

\[ 3|(4^n + 2) \text{ for all integers } n \geq 1. \]

4. \( N \) balls are distributed into 3 boxes. The smallest value of \( N \) that guarantees that either the first box contains at least 4 balls or the second box contains at least 5 balls or the third box contains at least 6 balls is:
5. Consider the English alphabet of 26 letters including the 5 vowels a, e, i, o, u.
   (a) The number of sets of 7 letters with exactly 3 vowels is:

   (b) The number of different sequences of 7 letters containing 7 distinct letters including exactly 3 vowels is:

6. Solve the recurrence relation

   \[ a_n = 2a_{n-1} + 3a_{n-2} \quad (n \geq 3), a_1 = 1, a_2 = 4. \]
7. An urn contains 5 balls of each of the colors R, W, and B (so 15 altogether). The numbers $-4, -1, 1, 2, 3$ are written on each the balls of each color (each number occurs once with each color). Suppose you reach into the urn and grab 3 balls all at once.

(a) The probability that all the ball have different colors is:

(b) The probability that all the balls have the same color is:

(c) Suppose one makes a game out of the above urn whereby drawing a ball pays you in dollars the number on the ball (negative numbers means you pay). If you play this game, say, a 100 times, you would expect to win how much per game?

8. The matrix of the transitive closure of the relation $R$ on \{1, 2, 3, 4, 5\} in which $1R2$, $1R3$, $1R5$, $2R3$, $3R4$, $4R2$, $5R1$ is
9. For each of the following two relations, determine whether they are reflexive, irreflexive, symmetric, antisymmetric, and transitive. If an equivalence relation, determine the partition into equivalence classes.

(a) \( R \) the relation whose matrix is

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 
\end{bmatrix}
\]

Reflexive ... Irreflexive ... Symmetric ... Antisymmetric ... Transitive

Equivalence classes (if applicable) are:

(b) the relation \( R \) on the set of integers \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\} \)
where \( aRb \) if and only if \( a = b \pm 1 \).

Reflexive ... Irreflexive ... Symmetric ... Antisymmetric ... Transitive

Equivalence classes (if applicable) are: