Complete the following items, staple this page to the front of your work, and turn your assignment in class on Thursday, September 25.

**Modular arithmetic**

1. Determine the last digit of $3^{400}$, then the last two digits. Determine the last digit of $7^{99}$.

2. Prove that there are infinitely many primes of the form $4n - 1$.

**Solving congruences**

3. Prove that if $x^2 \equiv n \pmod{65}$ has a solution, then so does $x^2 \equiv -n \pmod{65}$.

4. Solve the following congruences:
   a. $6x + 3 \equiv 1 \pmod{10}$
   b. $15x \equiv 25 \pmod{35}$
   c. Simultaneously: $x \equiv 1 \pmod{4}$, $x \equiv 7 \pmod{13}$
   d. Simultaneously: $x \equiv 11 \pmod{142}$, $x \equiv 25 \pmod{86}$

**Equivalence relations**

5. Define a relation on $\mathbb{R}$ as follows: $x \sim y$ if and only if $x - y$ is an integer. Prove that $\sim$ is an equivalence relation and describe the set of equivalence classes.

6. Given a function $f : S \rightarrow T$, consider the following relation on $S$: $x \sim y \iff f(x) = f(y)$.
   a. Prove that $\sim$ is an equivalence relation.
   b. Prove that if $f$ maps onto $T$, then there is a one-to-one correspondence between the set of equivalence classes and $T$.

I discussed these exercises with the following people: