1. Find the center, radius and $x$-intercept(s) of the circle

$$18x^2 + 18y^2 - 12x + 9y - 7 = 0$$

Solution

$$18x^2 + 18y^2 - 12x + 9y - 7 = 0$$

$$x^2 + y^2 - \frac{12}{18}x + \frac{9}{18}y = \frac{7}{18}$$

$$x^2 - \frac{2}{3}x + y^2 + \frac{1}{2}y = \frac{7}{18}$$

$$x^2 - \frac{2}{3}x + \frac{1}{9} + y^2 + \frac{1}{2}y + \frac{1}{16} = \frac{7}{18} + \frac{1}{9} + \frac{1}{16}$$

$$\left( x - \frac{1}{3} \right)^2 + \left( y + \frac{1}{4} \right)^2 = \frac{81}{144}$$

$$\left( x - \frac{1}{3} \right)^2 + \left( y + \frac{1}{4} \right)^2 = \left( \frac{9}{12} \right)^2$$

$$\left( x - \frac{1}{3} \right)^2 + \left( y + \frac{1}{4} \right)^2 = \left( \frac{3}{4} \right)^2$$

So the center is $\left( \frac{1}{3}, -\frac{1}{4} \right)$ and the radius is $3/4$. To find the $x$-intercept we set $y = 0$:
\[
\left( x - \frac{1}{3} \right)^2 + \left( 0 + \frac{1}{4} \right)^2 = \left( \frac{3}{4} \right)^2 \\
\left( x - \frac{1}{3} \right)^2 + \frac{1}{16} = \frac{9}{16} \\
\left( x - \frac{1}{3} \right)^2 = \frac{8}{16} = \frac{1}{2} \\
x - \frac{1}{3} = \pm \sqrt{\frac{1}{2}} \\
x = \frac{1}{3} \pm \frac{1}{\sqrt{2}}
\]

2. Solve the inequality \( \frac{2 - 3x}{x + 4} \leq 0 \)

Solution

We first find when the numerator and denominator are zero:

\[
2 - 3x = 0 \iff x = \frac{3}{2} \\
x + 4 = 0 \iff x = -4
\]

Hence the real line is split in the intervals \((-\infty, -4), (-4, 3/2)\) and \((3/2, \infty)\).

Using some numbers (or otherwise) we see that the expression \( \frac{2 - 3x}{x + 4} \) is positive on \((-4, 3/2)\) and negative on \((-\infty, -4)\) and \((3/2, \infty)\). We also know that the expression is undefined when \(x = -4\) and it is equal to zero when \(x = 3/2\). Since we need to find when the expression is less than or equal to zero, the solution is \((-\infty, -4) \cup [3/2, \infty)\).