1. Let \( f(x) = \sqrt{10-x} \) and \( g(x) = \frac{1}{x-7} \). Compute \((g \circ f)(x)\) and find its domain.

Solution

\[
(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{10-x} - 7}
\]

For the domain we need the quantity under the square root to be nonnegative, so

\[
10 - x \geq 0 \iff x \leq 10.
\]

We also need the denominator of the fraction to be different than zero. But

\[
\begin{align*}
\sqrt{10-x} - 7 &= 0 \\
\sqrt{10-x} &= 7 \\
10 - x &= 49 \\
x &= -39
\end{align*}
\]

so we need to have \( x \neq -39 \) along with \( x \leq 10 \). Therefore the domain is \((-\infty, -39) \cup (-39, 10]\).
2. Let \( f(x) = -x^2(x - 1)(x + 3)^2 \).

(a) Determine the left/right-end behavior of \( f \).
(b) Find the \( x \)-intercept(s).

**Solution**

(a) We begin by finding the degree of the polynomial, which is the sum of the multiplicities: \( 2 + 1 + 2 = 5 \). Hence the degree is an odd number, which means that the left-end and right-end behavior of \( f \) are going to be opposite of each other. Next we see that the leading coefficient is \(-1\), which is negative. Therefore the left-end behavior is ”up” (the function is positive and decreasing) and the right-end behavior is ”down” (the function is negative and decreasing).

\[
-x^2(x - 1)(x + 3)^2 = 0
\]
\[
x^2 = 0, \quad x - 1 = 0, \quad (x + 3)^2 = 0
\]
\[
x = 0, \quad x = 1, \quad x = -3
\]

3. Find all the roots of the polynomial \( f(x) = x^3 - 2x^2 - 2x - 3 \).

**Solution**

We start by trying to identify one root. Since the constant term is equal to \(-3\) and the leading coefficient is equal to 1, the only possible rational roots for \( f(x) \) are \( \pm 1 \) and \( \pm 3 \). After some arithmetic we see that \( f(1) \neq 0, f(-1) \neq 0 \), but \( f(3) = 0 \), so \( x = 3 \) is a root. Therefore \( x - 3 \) divides \( f(x) \) and we can use synthetic division (or long division) to find that the quotient is \( q(x) = x^2 + x + 1 \), that is

\[
f(x) = (x - 3)(x^2 + x + 1).
\]

The remaining roots of \( f(x) \) are precisely the roots of \( x^2 + x + 1 \). Using the quadratic formula, we see that these are \(-\frac{1}{2} \pm \frac{\sqrt{3}}{2} i\).