1. Compute the second order Taylor polynomial of $e^{x+2y}$ at $(1,0)$.

\[
\begin{align*}
f(x,y) &= e^{x+2y}, \quad f_x(x,y) = e^{x+2y}, \quad f_y(x,y) = 2e^{x+2y} \\
f_{xx}(x,y) &= e^{x+2y}, \quad f_{xy}(x,y) = 2e^{x+2y}, \quad f_{yy}(x,y) = 4e^{x+2y}
\end{align*}
\]

\[
Q(x,y) = f(1,0) + f_x(1,0)(x-1) + f_y(1,0)y \\
+ \frac{1}{2} \left( f_{xx}(1,0)(x-1)^2 + 2f_{xy}(1,0)(x-1)y + f_{yy}(1,0)y^2 \right)
\]

\[
= e + e(x-1) + 2ey + \frac{1}{2} \left( e(x-1)^2 + 4e(x-1)y + 4ey^2 \right)
\]

2. Find all critical points of

\[
f(x,y) = x^2 - 4xy + y^2 - y.
\]

Apply the second derivative test to all critical points to determine if each critical point is a local maximum, or a local minimum, or none of the two.

\[
f_x(x,y) = 2x - 4y \quad f_y(x,y) = -4x + 2y - 1
\]

\[
f_x(x,y) = 0 \implies 2x - 4y = 0 \quad x = 2y
\]

\[
f_y(x,y) = 0 \implies -4x + 2y - 1 = 0 \implies 8y - 2y - 1 = 0 \implies y = \frac{1}{6}
\]

\[
f_{xx}(x,y) = 2, \quad f_{xy}(x,y) = -4, \quad f_{yy}(x,y) = 2
\]

\[
Hf(-\frac{1}{3}, -\frac{1}{6}) = f_{xx}(-\frac{1}{3}, -\frac{1}{6})f_{yy}(-\frac{1}{3}, -\frac{1}{6}) - (f_{xy}(-\frac{1}{3}, -\frac{1}{6}))^2
\]

\[
= 2 \cdot 2 - (-4)^2 = -12 < 0
\]

so $f$ has a saddle point at $\left(-\frac{1}{3}, -\frac{1}{6}\right)$