1. (5 points) Let \( \mathbf{r}(t) = (\cos 4t) \mathbf{i} + (\sin 4t) \mathbf{j} + 3t \mathbf{k} \), be the position of a particle in space at time \( t \) \( (-\infty < t < \infty) \).

(a) Compute the velocity and speed of the particle, at any time \( t \).

(b) Find an equation for the line tangent to the curve \( \mathbf{r}(t) \), when \( t = 0 \).

\[
\mathbf{v}'(t) = \mathbf{r}'(t) = -4 \sin 4t \, \mathbf{i} + 4 \cos 4t \, \mathbf{j} + 3 \, \mathbf{k}
\]

speed: \( |\mathbf{v}'(t)| = \sqrt{(-4 \sin 4t)^2 + (4 \cos 4t)^2 + 3^2} \)

\[
= \sqrt{16 (\sin^2 4t + \cos^2 4t) + 9} = \sqrt{16 + 9} = \sqrt{25} = 5
\]

b) \( \mathbf{r}'(0) = \cos 0 \, \mathbf{i} + \sin 0 \, \mathbf{j} + 3 \cdot 0 \, \mathbf{k} = \mathbf{v} \)

\[
\mathbf{v}'(0) = -4 \sin 0 \, \mathbf{i} + 4 \cos 0 \, \mathbf{j} + 3 \, \mathbf{k} = 4 \, \mathbf{j} + 3 \, \mathbf{k}
\]

\[
\mathbf{r}(s) = \mathbf{r}(0) + s \mathbf{v}'(0) = \mathbf{v}' + s \left( 4 \mathbf{j} + 3 \mathbf{k} \right) \quad \text{vector equation}
\]

or \[
\begin{align*}
x &= s \\
y &= 4s \\
z &= 3s \\
\end{align*}
\quad \text{cartesian parametric equations}
\]
2. (5 points) A baseball is thrown from the stands 32 ft above the field at an angle of 30° up from the horizontal and with initial speed of 32 ft/sec.

(a) Use the above data to write a vector equation for the trajectory of the baseball (remember \( g = 32 \text{ ft/sec}^2 \)).

(b) When is the ball going to hit the ground? How far is it going to be then?

\[
\vec{r}(t) = (v_0 \cos 30) \hat{i} t \hat{i} + \left[ y_0 + (v_0 \sin 30) t - \frac{1}{2} g t^2 \right] \hat{j} \\
= (32 \cos 30) t \hat{i} + \left[ 32 + (32 \sin 30) t - \frac{1}{2} 32 t^2 \right] \hat{j} \\
= 16\sqrt{3} t \hat{i} + \left( -162 + 16t + 32 \right) \hat{j}
\]

\[
y(t) = 0 \Rightarrow -16t^2 + 16t + 32 = 0 \\
\Rightarrow -16(t^2 - t - 2) = 0 \\
\Rightarrow -16(t + 1)(t - 2) = 0 \\
\Rightarrow t = -1 \text{ or } t = 2 \\
\Rightarrow t = 2 \text{ since } t > 0
\]

\[
x(2) = 16\sqrt{3} \cdot 2 = 32\sqrt{3}
\]

So the ball hits the ground 2 seconds later and it is \( 32\sqrt{3} \) ft far at that time.