1. (5 points) Sketch the region of integration for the following integral. Then convert to polar coordinates and compute.

\[
\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{1}{x^2 + y^2 + 1} \, dx \, dy
\]

\[
r^2 = x^2 + y^2
\]

\[
= \int_0^{2\pi} \int_0^1 \frac{r}{r^2 + 1} \, r \, dr \, d\theta
\]

\[
u = r^2 + 1 \quad \Rightarrow \quad du = 2r \, dr
\]

\[
\Rightarrow \quad r = 0 \Rightarrow u = 1 \quad \text{and} \quad r = 1 \Rightarrow u = 2
\]

\[
= \int_0^{2\pi} \frac{1}{2} \ln(u) \bigg|_u=1^u=2 \, d\theta
\]

\[
= \int_0^{2\pi} \frac{1}{2} \left( \ln 2 - \ln 1 \right) \, d\theta
\]

\[
= \frac{1}{2} \cdot 2\pi \ln 2 = \pi \ln 2
\]
2. (5 points) Compute the volume of the region cut from the cylinder $x^2 + y^2 = 4$, by the planes $z = 0$ and $x + z = 3$.

\[ V = \int_{-2}^{2} \int_{\sqrt{-y^2}}^{\sqrt{4-y^2}} \int_{3-x}^{3-x} dz \, dx \, dy \]

\[ = \int_{-2}^{2} \int_{\sqrt{-y^2}}^{\sqrt{4-y^2}} (3-x) \, dx \, dy \]

\[ = \int_{-2}^{2} \left[ 3x - \frac{x^2}{2} \right]_{x=-\sqrt{4-y^2}}^{x=\sqrt{4-y^2}} dy \]

\[ = \int_{-2}^{2} \left( 3\sqrt{4-y^2} - 4 + \frac{4}{2} + 3\sqrt{4-y^2} - \frac{4-y^2}{2} \right) dy \]

\[ = 6 \int_{-2}^{2} \sqrt{4-y^2} \, dy = 6 \cdot 2\pi = 12\pi \]

since $\int_{-2}^{2} \sqrt{4-y^2} \, dy$ gives the area of a semicircle of radius 2, so it is equal to $2\pi$.

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**Note 1:** We could have computed $\int_{-2}^{2} \sqrt{4-y^2} \, dy$ using the trig substitution $y = 2\sin\theta$.

**Note 2:** We could have computed $\int_{-2}^{2} \int_{\sqrt{-y^2}}^{\sqrt{4-y^2}} 3-x \, dx \, dy$ using polar coordinates. We would get the integral $\int_{0}^{2\pi} \int_{0}^{2} (3-r\cos\theta) r \, dr \, d\theta$, which can be computed easily.