Fall 2012
Midterm Exam I

1. (a)

(b) \( \lim_{(x,y) \to (0,0)} f(x,y) \) does not exist.

(c) No, because \( f \) is not continuous at the origin.

2. (a)

\[
\begin{align*}
g_x &= 1 + ye^{xy}, \\
g_y &= -1 + xe^{xy}, \\
g_{xx} &= y^2 e^{xy}, \\
g_{yy} &= x^2 e^{xy}, \\
g_{xy} &= g_{yx} = (1 + xy)e^{xy}.
\end{align*}
\]

(b) \( \vec{r}(t) = t \hat{i} + 2 \hat{j} + (3 + 4t) \hat{k} \).

3. (a)

\[
\begin{align*}
\frac{\partial z}{\partial x} &= \frac{1 + yz \cos(xyz)}{3z^2 + xy \cos(xyz)}, \\
\frac{\partial z}{\partial y} &= \frac{-2y + xz \cos(xyz)}{3z^2 + xy \cos(xyz)}.
\end{align*}
\]

(b) \( (x - 2) + 2y + 3(z - 1) = 0 \).
4. (a) The direction of most rapid increase is \((\hat{i} - 2\hat{j})/\sqrt{5}\), in which the rate of change is \(2\sqrt{5}\).
   (b) \(2/\sqrt{10}\).
   (c) \(L(x, y) = 3\ln 3 + 2(x - 1) - 4(y + 1)\).

Final Exam

1. \(\vec{r}_l(\tau) = (2 - \tau)\hat{i} + (1 + 3\tau)\hat{j} + (1 - 12\pi\tau)\hat{k}\).

Fall 2013

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1. (a) \(\lim_{(x, y) \to (0, 0)} f(x, y) = -1\).
   (b) \(c = -1\).

2. • \(g^{-1}(1)\) is the line \(y = x\) (red), while \(g^{-1}(3)\) consists of the lines \(y = x \pm 2\) (blue).
   • \(\nabla g(3, 1) = 2\hat{i} - 2\hat{j}\) (green), and \(\nabla g(-2, 0) = -2\hat{i} + 2\hat{j}\) (yellow).
3. (a)
\[
\frac{\partial z}{\partial x} = -\frac{y^2z + 3(x + 4y + 2z)^2}{xy^2 + 6(x + 4y + 2z)^2},
\frac{\partial z}{\partial y} = -\frac{2xyz + 12(x + 4y + 2z)^2}{xy^2 + 6(x + 4y + 2z)^2}.
\]
(b) \( \vec{r}(t) = (2 + t) \hat{i} + (-1 - 4t) \hat{j} + (1 + 2t) \hat{k} \).

4. (a) \(-12/\sqrt{5}\).
(b) The direction of most rapid decrease is \((\hat{i} - \hat{j})/\sqrt{2}\), in which the rate of change is \(-4\sqrt{2}\).

5. (a)
\[
\begin{align*}
&h_x = e^{\sin y}, &h_{xx} = 0, \\
&h_y = xe^{\sin y} \cos y, &h_{yy} = xe^{\sin y} \cos^2 y - xe^{\sin y} \sin y, \\
&h_{xy} = h_{yx} = e^{\sin y} \cos y. &h_{yx} = h_{yy} = e^{\sin y} \cos y.
\end{align*}
\]
(b) \( L(x, y) = -x + 3y - 3\pi \).
(c) \(-3.56\).

Final Exam
1. 0.