Name: Evangelos Dimou

Note: Show all your work. No calculators allowed.

1. (5 points) Find the angle between the lines $3x - y = -5$ and $y = -2x + 7$.

Solution

The angle between the two lines will be equal to the angle between the corresponding normal vectors, so it suffices to find the latter.

$n_1 = 3\mathbf{i} - \mathbf{j}, \quad n_2 = 2\mathbf{i} + \mathbf{j}$

$n_1 \cdot n_2 = 3 \cdot 2 + (-1) \cdot 1 = 5$

$|n_1| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

$|n_2| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\cos \theta = \frac{n_1 \cdot n_2}{|n_1||n_2|} = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}, \text{ so } \theta = \frac{\pi}{4}$
2. (5 points) Find the equation of the plane that goes through the points $A(2, 1, -1)$, $B(3, 0, 1)$ and $C(1, -2, 0)$.

Solution

$\overrightarrow{AB} = (3 - 2)i + (0 - 1)j + (1 - (-1))k = i - j + 2k$

$\overrightarrow{AC} = (1 - 2)i + (-2 - 1)j + (0 - (-1))k = -i - 3j + k$

\[ \vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -1 & -3 & 1 \end{vmatrix} = 5i - 3j - 4k \]

If $P(x, y, z)$ is a point on the plane then $\vec{n} \perp \overrightarrow{AP}$, so

\[ \vec{n} \cdot \overrightarrow{AP} = 0 \iff [5i - 3j - 4k] \cdot [(x - 2)i + (y - 1)j + (z + 1)k] = 0 \]

\[ \iff 5(x - 2) - 3(y - 1) - 4(z + 1) = 0 \]

\[ \iff 5x - 3y - 4z = 11 \]