1. Consider the region $D$ in the first octant bounded above by $x^2 + y^2 + z^2 = 4$ and below by $z = \sqrt{x^2 + y^2}$.

   (a) (5 points) Set up an integral for the volume of $D$ using cylindrical coordinates. Show enough work to justify how you got the bounds for each integration variable, including all necessary sketches. Do not compute the integral.

Solution

$$V = \iiint_D 1 \, dV = \int_0^{\pi/2} \int_0^{\sqrt{4-r^2}} \int_r^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$$
(b) (5 points) Set up an integral for the volume of $D$ using spherical coordinates. Show enough work to justify how you got the bounds for each integration variable, including all necessary sketches. Do not compute the integral.

Solution

\[ V = \iiint_D 1 \, dV = \int_0^{\pi/2} \int_0^{\pi/4} \int_0^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \]