Math 321  Quiz 3W  October 19, 2011

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Note: Show all your work. No calculators allowed.

1. (a) (1.5 point) Give the implicit cartesian equation of a hyperbola in the xy-plane, having its two vertices on the x-axis. Make a sketch.

   \[ \frac{(x-x_c)^2}{a^2} - \frac{y^2}{b^2} = 1 \]

(b) (1.5 point) Give a parametric cartesian equation of the above hyperbola.

   a) \[ \frac{(x-x_c)^2}{a^2} - \frac{y^2}{b^2} = 1 \]

   b) Since \( \sec^2 \theta - \tan^2 \theta = 1 \)
we write \( \frac{x-x_c}{a} = \sec \theta \) and \( \frac{y}{b} = \tan \theta \)

   or \( x = x_c + a \sec \theta \)
   \( y = b \tan \theta \)

2. (2 points) In this problem all vectors are functions of \( t \).

   (a) Is the following statement true or false? (No justification needed)

   \[ \frac{d}{dt} (\vec{r} \times \vec{\dot{v}}) = \vec{r} \times \frac{d\vec{\dot{v}}}{dt}, \text{ where } \vec{\dot{v}} = \frac{d\vec{r}}{dt} \]

   (b) Complete the identity (No proof needed)

   \[ \frac{d}{dt} [(\vec{a} \times \vec{b}) \cdot \vec{c}] = (\frac{d\vec{a}}{dt} \times \vec{b}) \cdot \vec{c} + (\vec{a} \times \frac{d\vec{b}}{dt}) \cdot \vec{c} + (\vec{a} \times \vec{b}) \cdot \frac{d\vec{c}}{dt} \]
3. (5 points) Consider the equation \( \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r} \), where \( \vec{\omega} \) is a constant vector, but \( \vec{r} \) depends on \( t \).

(a) Prove that \( \vec{r} \) has constant magnitude. Show all steps.
(b) Show that \( \vec{\omega} \cdot \vec{r} \) is constant.
(c) Show that \( \theta \), the angle between \( \vec{\omega} \) and \( \vec{r} \), is also constant.
(d) Conclude that \( \vec{v} = \frac{d\vec{r}}{dt} \) has constant magnitude.

See solution of Quiz 3M.