1. (3 points) Compute $\nabla f$, where $f(\hat{r}) = \frac{1}{|\hat{r} \times \hat{r}|}$.

$$|\hat{r} \times \hat{r}| = |\hat{r}|^2 \sin \Theta = r \sin \Theta = \rho$$

$$f = f(\rho) = \frac{1}{\rho}$$

so

$$\nabla f = \frac{\partial f}{\partial \rho} \nabla \rho$$

$$\Rightarrow \nabla f = -\frac{1}{\rho^2} \hat{\rho}$$

Of course we can write

a) $f = f(r, \theta) = \frac{1}{r \sin \theta}$

and then compute $\nabla f$ in spherical coordinates, or

b) $f = f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$

and then compute $\nabla f$ in cartesian coordinates.

Try to verify that all 3 approaches give the same result.

2. (a) (1 point) Express $\nabla f$ in cylindrical coordinates:

$$\nabla f = \frac{\partial f}{\partial \rho} \nabla \rho + \frac{\partial f}{\partial \phi} \nabla \phi + \frac{\partial f}{\partial z} \nabla z$$

$$= \frac{\partial f}{\partial \rho} \hat{\rho} + \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

(b) (1 point) Express $\nabla \cdot \vec{F}$ in cartesian coordinates:

$$\nabla \cdot \vec{F} = \left( \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) \cdot (F_x \hat{x} + F_y \hat{y} + F_z \hat{z})$$

$$= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$
3. (5 points)

(a) Write $\hat{\rho}$ and $\hat{\phi}$ in hybrid representation.
(Hint: Make a 2D sketch showing the relationship between $\hat{\rho}$, $\hat{\phi}$
and $\hat{x}$, $\hat{y}$.)

(b) Compute $\frac{\partial \hat{\rho}}{\partial \varphi}$ and $\frac{\partial \hat{\phi}}{\partial \varphi}$ and then express those quantities in cylindrical coordinates.

See solution of Quiz 6W