(1) Suppose $R$ is a commutative ring with 1. Suppose $K$ and $L$ are ideals of $R$ (so that they are $R$-submodules). Show that
\[ R/K \cong R/L \text{ as } R\text{-modules} \iff K = L \]

(2) Suppose that $R$ is a PID and $M$ a cyclic module over $R$. Show that any submodule of $M$ is cyclic. Hint: Use the lattice isomorphism theorem for modules and the structure theorem for cyclic modules.

(3) A column finite matrix is a matrix with infinitely many rows and columns so that each column contains only finitely many nonzero entries. Let $R$ be the ring of all column finite matrices over a field $F$ (with the usual operations of matrix addition and multiplication). Let $M = R$ as an $R$ module. Find a basis of $M$ consisting of 1 element and another basis for $M$ consisting of 2 elements.

(4) Let $R = M_{2 \times 2}(F)$ be the ring of $2 \times 2$ matrices over a field $F$. Let $M = F^2$ (with elements regarded as column matrices). $M$ is an $R$-module with the usual addition and action by left multiplication. Let $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in M$.

(a) Show that $M$ is a cyclic module with generator $e_1$.
(b) Find $Ann(M)$ and $Ann(e_1)$.
(c) Is $Ann(e_1)$ an ideal of $R$?

(5) Suppose that $M$ is an $R$-module and $x$ is an element of $M$ so that $Ann(x) = \{0\}$. Suppose that $a \in R$. Show that
\[ Rx/Rax \cong R/(a). \]
Indicate where you use the fact that $Ann(x) = \{0\}$. 