(1) Suppose that $E/F$ is an extension so that $[E:F] = 1$. Show that $E = F$.

(2) Let $u = \cos(\pi/9) \in \mathbb{C}$. Show that $u$ is algebraic over $\mathbb{Q}$ and find the minimal polynomial of $u$ over $\mathbb{Q}$.

(3) (a) If $F$ is a field and $p = \text{char}(F)$ (see notes) show that either $p$ is 0 or $p$ is a prime.
   (b) If $E$ is an extension of $\mathbb{Q}$ what is $\text{char}(E)$?
   (c) Let $p$ be a prime and let $\mathbb{F}_p = \mathbb{Z}/(p) = \{0, 1, \ldots, p-1\}$ be the field of integers mod $p$. What is $\text{char}(\mathbb{F}_p)$?

(4) Let $r = \sqrt[3]{5}$. Find the inverse of $1 + r - r^2$ in $\mathbb{Q}(r)$. Express your answer in the form $a_0 + a_1r + a_2r^2$ where $a_0, a_1, a_2 \in \mathbb{Q}$.

(5) Suppose that $E/F$ is an extension. Let $K$ be the set of all elements of $E$ which are algebraic over $F$. Show that $K$ is a subfield of $E$. (In the case when $E/F$ is $\mathbb{C}/\mathbb{Q}$, $K$ is called the field of algebraic numbers.)