Assume $U$ is connected. A function $u \in H^1(U)$ is a weak solution of Neumann’s problem
\[
\begin{cases}
-\Delta u = f & \text{in } U \\
\frac{\partial u}{\partial \nu} = 0 & \text{on } \partial U
\end{cases}
\tag{1}
\]
if
\[
\int_U Du \cdot Dv dx = \int_U fv dx
\]
for all $v \in H^1(U)$. Suppose $f \in L^2(U)$. Prove (1) has a weak solution if and only if
\[
\int_U f dx = 0.
\]