Recall that we learned yesterday that

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

and

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}.$$  

1. Use implicit differentiation to find the derivative of $f(x) = \text{arcsec}(x)$.

2. Find $\frac{dy}{dx}$ for the following curves. (Your answer will typically be in terms of both $x$ and $y$.)

   (a) $x^2 - y^2 = x$
   (b) $2\sqrt{x} + \sqrt{y} = 3$
   (c) $x^2 + xy - y^2 = 4$
   (d) $y^5 + 3x^2y^2 + 5x^4 = 12$
   (e) $y \cos(x) = x^2 + y^2$
   (f) $\cos(xy) = 1 + \sin(y)$

3. Consider the curve $x^3 + y^3 = 6xy$. It is called a “folium of Descartes.”

   (a) Find $\frac{dy}{dx}$.
   (b) Find the slope of the tangent line at the point (3, 3).
   (c) Find the slope of the tangent line at the point (0, 0)?

4. Find $\frac{d^2y}{dx^2}$ (that is, the second derivative) if $x^2 + y^2 = 16$. (Hint: first find $\frac{dy}{dx}$. Then take the derivative of your answer with respect to $x$.)

5.* Suppose $y = f(x)$ is a nice, smooth function, and suppose $f(x)$ is also one-to-one, so it has an inverse. Assume that $f(3) = 4$, and $f'(3) = 2$.

   (a) Draw a picture of what $f(x)$ might look like. Make sure your picture reflects your knowledge of the slope of the tangent line at the point (3, 4), and that $f(x)$ is one-to-one.
   (b) Draw a second picture of what $y = f^{-1}(x)$ might look like. You know $y = f(x)$ passes through the point (3, 4). What point do you know $y = f^{-1}(x)$ passes through?
   (c) What is the slope of the tangent line at the point (4, 3) on the graph of $y = f^{-1}(x)$? (Hint: how did we find the derivative of $\arcsin(x)$, just by using our knowledge of $\sin(x)$)?

6.* If you have a nice function $f(x)$ and you know it’s derivative, how can you take the derivative of it’s inverse? If you know the slope of the tangent line to the graph of $y = f(x)$ at the point $(a, b)$ (where $b = f(a)$), how do you find the slope of the tangent line to the graph of $y = f^{-1}(x)$ at the point $(b, a)$?

Questions with a * are more difficult, and designed to make you think, and improve your understanding. Questions without a * are good practice for quizzes/exams.