Steps for solving related rates problems:

a. Draw a picture of the situation at any time $t$ and name your variables. You should label quantities that are changing as variables.

b. What rates are known? What rates are unknown? What is constant? (The first two steps are done together.)

c. Write an equation that relates the variables— it should involve the variable(s) you know the rate(s) of change of and the variable(s) you seek the rate(s) of change of.

d. Simplify the equation if possible, and differentiate both sides with respect to time.

e. Plug in the given information and finish solving the problem. Remember not to plug in “frozen information” until this step.

Do the following problems on clean paper so you can write out all the steps clearly.

0. (from yesterday’s lecture) A 10-foot ladder is resting against a wall (the wall is vertical and the floor is horizontal). The bottom end of the ladder is sliding away from the wall at a constant speed of 1 foot per second. At what speed is the top end of the ladder sliding down the wall at the moment when the bottom end of the ladder is 6 feet away from the wall?

1. A plane flies directly over Van Vleck Hall while traveling at a constant altitude 1 mile and a constant speed of 400 miles per hour. Find the rate at which the distance from the plane to Van Vleck Hall is increasing when the plane is 3 miles away from the building. (Here distance means total distance, not horizontal distance.)

2. Thor starts walking north at a constant rate of 6 feet per second. Three minutes later, Loki starts walking south from a point 400 feet west of where Thor started his journey, moving at 4 feet per second. How fast are they moving apart 10 minutes after Loki started walking?

3. A tank is shaped like an inverted cone (so the point is at the bottom). The circular top of the cone has a radius of 4 feet and the height of the cone is 3 feet. Water is pouring into the tank at a constant rate of 1 cubic foot per minute. At what rate is the height of the water rising when the water is 2 feet high?

4. A particle moves along the curve $y = \sqrt{x}$. As it passes through the point $(9, 3)$, its $x$-coordinate is increasing at a rate of 2 meters per second. At what rate is the particle’s distance to the origin changing at this moment?

5. A baseball diamond is a 90 foot by 90 foot square. Michael Jordan hits the ball and runs toward first base. When he is halfway to first base, he is running 24 feet per second. At what rate is the distance between M.J. and second base decreasing at this instant? At what rate is the distance between M.J. and third base increasing?

6. Fitz and Simmons are riding on a Ferris wheel whose center is 30 feet above the ground and radius is 24 feet. The wheel completes one full turn per minute. At what rate is their altitude rising when they are 42 feet above the ground (and going up)?
7. * Harry, Ron, and Hermione are standing against a wall. Ron is in the middle, Harry is one meter to Ron’s left, and Hermione is 1 meter to Ron’s right. Harry and Hermione are holding opposite ends of a magical stretchy rope, and Ron is holding the middle of the rope. Harry and Hermione start walking at the same time straight away from the wall (so they are walking parallel to each other), but Ron stays still, so that the rope makes an angle with Ron at the vertex. Harry is walking at a constant speed of 4 feet per second, while Hermione walks at a rate of 2 feet per second. At what rate is the angle decreasing when its measure is $\pi/6$ radians?