1. Prove the parallelogram law: if $x$ and $y$ are two vectors in an inner product space $V$ and $\| \cdot \|$ is the norm induced by the inner product $\langle \cdot, \cdot \rangle$, then

\[
\| x + y \|^2 + \| x - y \|^2 = 2\| x \|^2 + 2\| y \|^2.
\]

2. Let $W = \text{Span}\{(1, 2, 0, 2), (2, -1, 2, 4), (0, 0, 4, 3)\} \subseteq \mathbb{R}^4$. Find an orthonormal basis for $W$ by the following procedure.

(a) Start with the first vector you were given, and call it $v_1$. Then replace the second vector by a new vector $v_2$ that is orthogonal to $v_1$ but such that $v_1$ and $v_2$ span the same subspace as the first two vectors you were given.

(b) Next, replace the third vector you were given by a vector $v_3$ orthogonal to both $v_1$ and $v_2$, but make sure you have the right span! (Think about orthogonal projections.)

(c) Finally, make sure all your vectors have length 1.

3. Let $S$ be a subset of an inner product space $V$, and let

\[
S^\perp = \{ x \in V \mid x \perp y \text{ for all } y \in S \}.
\]

(a) Prove that $S^\perp$ is a subspace of $V$.

(b) If $V = \mathbb{R}^3$ and $S = \{(1, 0, 0)\}$, then what does $S^\perp$ look like?

(c) If $W$ is a subspace of $V$, what do you think the dimension of $W^\perp$ should be, in terms of the dimensions of $V$ and $W$?