Section 5.5 Problems 5,6,7 In each of the problems 5,6,7
(a) Show that the given differential equation has a regular singular point at \( x = 0 \)
(b) Determine the indicial equation, the recurrence relation, and the roots of the indicial equation
(c) Find the series solution \((x > 0)\) corresponding to the larger root
(d) If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.
5. \(3x^2y'' + 2xy' + x^2y = 0\)
6. \(x^2y'' + xy' + (x-2)y = 0\)
7. \(xy'' + (1-x)y' - y = 0\)

Section 5.5 Problem 14 The Bessel equation of order zero is
\[ x^2y'' + xy' + x^2y = 0. \]
(a) Shows that \(x = 0\) is a regular singular point.
(b) show that the roots of the indicial equation are \(r_1 = r_2 = 0\).
(c) Show that one solution for \(x > 0\) is
\[ J_0(x) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^n (n!)^2} \]
(d) Show that the series of \(J_0(x)\) converges for all \(x\). The function \(J_0\) is known as the Bessel function of the first kind of order zero.

Section 5.5 Problem 15 Referring Problem 14, use the method of reduction of order to show that the second solution of the Bessel equation of order zero contains a logarithmic term.

Hint: If \(y_2(x) = J_0(x)v(x)\), then
\[ y_2(x) = J_0(x) \int \frac{dx}{x|J_0(x)|^2}. \]
Find the first term in the series expansion of \(\frac{1}{x|J_0(x)|^2}\).