**Problem 1 (3 points):** Find the function \( y = f(x) \), which satisfies that \( y \) is the smallest of all possible solutions of \( y^2 = xy + 2x^2 \).

1) **Solution:** Since \( y \) is the solution of \( y^2 = xy + 2x^2 \), we need to solve this quadratic polynomial \( y^2 - xy - 2x^2 = 0 \) for \( y \).

By the quadratic formula, we find \( y = \frac{x \pm \sqrt{x^2 + 8x^2}}{2} = \frac{x \pm |3x|}{2} \).

Since \( y \) is the smallest solution, \( y = \frac{x - |3x|}{2} \).

**Problem 2 (4 points):** Let \( f(x) = \sin(2x + \frac{\pi}{2}) \), where the domain is \( a \leq x \leq a + \frac{\pi}{2} \).

a. Now let \( g(x) = f(2x) \). Calculate \( g(\pi) \).

b. Find a number \( a \) (there are multiple choices of \( a \), you just need to find one) such that \( f(x) \) has an inverse.

2) **Solution:** a.

\[ g(\pi) = f(2\pi) = \sin(4\pi + \frac{\pi}{2}) = 1. \]

b.

We can think the question backwards.

Let’s first try to find the inverse function for \( x \).

Apply \( \arcsin \) function to both side of the function \( y = \sin(2x + \frac{\pi}{2}) \), we find:

\[ x = \frac{1}{2} \arcsin(y) - \frac{\pi}{4}. \]

Since the domain for \( x \) is: \( a \leq x \leq a + \frac{\pi}{2} \), \( a \) is the smallest value of \( x \). And The smallest value of \( \arcsin(y) \) is \( -\frac{\pi}{2} \), we find the smallest value of \( x \) is: \( -\frac{\pi}{4} - \frac{\pi}{4} = -\frac{\pi}{2} \). Thus we can choose \( a \) to be \( -\frac{\pi}{2} \).

**Problem 3 (4 points):** For which values of \( m \) does the graph of \( y = m \cdot x \) intersect the graph of \( y = x^2 + 1 \) in exactly one point?

3) **Solution:** The intersection points correspond with the common solution of \( y = m \cdot x \), \( y = x^2 + 1 \).

That is to solve \( x^2 - mx + 1 = 0 \). By quadratic formula, we find:

\[ x = \frac{m \pm \sqrt{m^2 - 4}}{2} \]

Since there is a unique solution, we need to have \( m^2 - 4 = 0 \). Thus \( m = 2, -2 \).