Quiz 2
MATH 222-004

Name: ________________________________

Problem 1. Find $\int \frac{x^2+1}{x^2-3x+2} \, dx$

Solution 1.

We have to notice that this isn’t a proper rational function so we have to do long division and divide $x^2 + 1$ by $x^2 - 3x + 2$. We then find:

$$\frac{x^2 + 1}{x^2 - 3x + 2} = 1 + \frac{3x - 1}{(x - 1)(x - 2)}$$

Now we can do partial fractions:

$$\frac{3x - 1}{(x - 1)(x - 2)} = \frac{A}{x - 1} + \frac{B}{x - 2}$$

Multiplying to get a common denominator and equating the numerators gives us $A(x - 2) + B(x - 1) = 3x - 1$. If we let $x = 2$ we find that $B = 5$ and letting $x = 1$ we find that $A = -2$. So we now have:

$$\int 1 + \frac{-2}{x - 1} + \frac{5}{x - 2} \, dx$$

Now we can integrate:

$$\int \frac{x^2 + 1}{x^2 - 3x + 2} \, dx = x - 2 \ln |x - 1| + 5 \ln |x - 2| + C$$

Problem 2. Compute $\int \frac{1}{\sqrt{9 - x^2}} \, dx$

Solution 2.

This is in the classic form $\frac{1}{\sqrt{a^2 - x^2}}$ so we make the substitution $x = 3 \sin(\theta)$ and so $dx = 3 \cos(\theta) \, d\theta$. Substituting we find:

$$\int \frac{1}{\sqrt{9 - x^2}} \, dx = \int \frac{3 \sin(\theta)}{3 \sin(\theta)} \, d\theta = \int 1 \, d\theta = \theta + C$$

Putting this back in terms of $x$ we find $\int \frac{1}{\sqrt{9 - x^2}} \, dx = \arcsin(x/3) + C$