Quotient 7
MATH 222-004

Name: ________________________________

For full credit please explain all of your answers. **No calculators** are allowed.

**Problem 1.** A philanthropist endows a chair. This means that she donates $1,000,000 to the university. The university invests the money (it earns compounded continuously interest). Denote the interest rate on the investment by \( r \) (e.g. if \( r=.06 \), then the investment earns interest at a rate of 6% compounded continuously) and the balance in the investment account at time \( t \) by \( B(t) \).

Write and solve a differential equation for \( B(t) \) using the initial condition.

**Solution 1.**

We’ve seen this a few times before. Whenever we have interest compounded continuously we know the final solution will be \( B(t) = 1000000e^{rt} \). The differential equation is \( \frac{dB}{dt} = rB \) with \( B(0) = 1000000 \) \( \Box \)

**Problem 2.** Compute \( T^0_0 f(x), T^1_0 f(x), T^2_0 f(x) \) for \( f(x) = \ln(x) \).

**Solution 2.**

By definition we know \( T^0_0 f(x) = f(e^2) = 2, T^1_0 f(x) = f(e^2) + f'(e^2)(x - e^2) = 2 + \frac{1}{e^2}(x - e^2) \). Finally,

\[
T^2_0 f(x) = f(e^2) + f'(e^2)(x - e^2) + \frac{f''(e^2)}{2!}(x - e^2)^2 = 2 + \frac{1}{e^2}(x - e^2) - \frac{1}{2e^4}(x - e^2)^2
\]

\( \Box \)