Determine the convergence or divergence of the following integrals without computing them. Then compute them explicitly.

Problem 1. \( \int_4^\infty \frac{1}{x - 2} \, dx \).

Problem 2. \( \int_1^\infty \frac{1}{x^3 - x} \, dx \).

Problem 3. \( \int_1^\infty \frac{dt}{1 + e^{2x}} \).

Problem 4. \( \int_0^\infty \frac{\sin(x^2)}{x^2} \, dx \). What happens at \( x = 0 \)?
Problem 5. Find a solution to the initial value problem:

\[
\frac{dy}{dx} = e^{y}x^3
\]

With initial value \( y(0) = 0 \).

Problem 6. Find a solution to the initial value problem:

\[
\frac{dy}{dx} = y\sqrt{y^2 - 1}\cos(x)
\]

With initial value \( y(0) = 1 \).

Problem 7. Find the general solution to the differential equation

\[
\frac{dy}{dx} = x^2 + y^2x^2
\]