Math 320 Exam Crib Sheet

1. **Integration by Parts Formula**

\[ \int u \, dv = uv - \int v \, du \]

Example:

\[ \int x \exp(x) \, dx = x \exp(x) - \int \exp(x) \, dx + C = x \exp(x) - \exp(x) + C \]

with \( u = x \), \( dv = \exp(x) \, dx \), \( du = dx \), and \( v = \exp(x) \).

2. **Example of Partial Fractions**

\[ \int \frac{5}{x^2 - 5x + 6} \, dx = \int \frac{5}{(x - 2)(x - 3)} \, dx \]

Let

\[ \frac{5}{(x - 2)(x - 3)} = \frac{A}{(x - 2)} + \frac{B}{(x - 3)} \]

\[ = \frac{A(x - 3) + B(x - 2)}{(x - 2)(x - 3)} \]

Therefore

\[ (A + B)x = 0 \quad \text{and} \quad -3A - 2B = 5. \]

Solving \( A + B = 0 \) and \(-3A - 2B = 5\) gives \( A = -5 \) and \( B = 5 \). So finally

\[ \int \frac{5}{x^2 - 5x + 6} \, dx = \int -\frac{5}{x - 2} \, dx + \int \frac{+5}{x - 3} \, dx = -5 \ln |x - 2| + 5 \ln |x - 3| + C. \]

3. **Exponentials and the Natural Logarithm**: All arguments of \( \ln \) are assumed greater than zero.

\[ \ln(1) = 0 \]

\[ \ln(a/b) = \ln(a) - \ln(b) \]

\[ \ln(ab) = \ln(a) + \ln(b) \]

\[ \ln(a^r) = r \ln(a) \]
\[
\int \frac{1}{u} du = \ln |u| + C, \quad u \neq 0
\]

\[
\exp(\ln(x)) = x
\]

\[
\ln(\exp(x)) = x
\]

\[
\exp(a + b) = \exp(a) \exp(b)
\]

\[
\exp(a - b) = \frac{\exp(a)}{\exp(b)}
\]

\[
\exp(ab) = (\exp(a))^b = (\exp(b))^a
\]

4. **Taylor Series for** \(f(x)\) **about the point** \(x = x_0\):

\[
f(x) = \sum_{n=0}^{\infty} \frac{d^n}{dx^n} f(x)|_{x=x_0} \frac{(x - x_0)^n}{n!}
\]