Practice Problems for Final Exam

1. Solve, and state the region of validity of the solution.

\[(x^2 - 4)y' + 2y = (x^2 - 4)(2 - x)^{-1/2}, \quad y(1) = 1 \]  \hspace{1cm} (1)

2. Find the general solution to

\[\frac{x^2}{2} y'' + 2xy' - \frac{7}{8} y = \frac{x^4}{2}, \quad x > 0. \]  \hspace{1cm} (2)

3. Note: You can do most of parts (b) and (c) without part (a).

(a) Find a basis for the subspace spanned by the columns of \(A\), where \(A\) is given by

\[A = \begin{bmatrix} 5 & -2 & 2 \\ 3 & -2 & 1 \\ -1 & 6 & 1 \end{bmatrix}. \]  \hspace{1cm} (3)

(b) Find \(b_3\) such that \(Ax = b\) has solutions, with \(A\) given above and \(b^T = [3, 4, b_3]\). Show that \(b\) with this value of \(b_3\) is a linear combination of the basis vectors found in (a).

(c) Solve \(Ax = b\) for \(b^T = [3, 4, b_3]\) and the value of \(b_3\) found in (b).

4. Solve, and express the solution in terms of real functions.

\[\frac{dx}{dt} = \begin{bmatrix} 3 & -5 \\ 1 & 1 \end{bmatrix} x, \quad x^T(\pi/4) = [2 \ 4] \]  \hspace{1cm} (4)

5. (a) Write down the form of the particular solution for the system:

\[\frac{dx}{dt} = \begin{bmatrix} -4 & -5 \\ 5 & 4 \end{bmatrix} x + \begin{bmatrix} \exp(-3t) \\ \cos(3t) \end{bmatrix} \]  \hspace{1cm} (5)

(b) Write down the equations for the vector coefficients of your particular solution. You do not need to solve these equations.

6. Find the general solution to

\[\frac{d^6 y}{dx^6} + 4 \frac{d^5 y}{dx^5} + 8 \frac{d^4 y}{dx^4} + 16 \frac{d^3 y}{dx^3} + 20 \frac{d^2 y}{dx^2} + 16 \frac{dy}{dx} + 16y = 0 \]  \hspace{1cm} (6)

given that the characteristic equation factors into \((r + 2)^2(r^2 + 2)^2 = 0\). Express your solution in terms of real functions. Give the range of validity of the solution.
7. Write the form of the solution (including homogeneous and particular parts) to the following ODEs. Do NOT solve for the coefficients!

\[ y'' - 2y' + 4y = \exp(x)[x \sin(\sqrt{3}x) + \exp(\sqrt{3}x)] \quad (7a) \]

\[ y'' - 6y' = x^2 + x \cosh(6x) \quad (7b) \]