1. (20 points)
(a) Find the general solution to
\[ 3y'' - 4y' = -\frac{4}{3}y. \] (1)

(b) For initial conditions \( y(3/2) = a, \ y'(3/2) = b, \) give a \( 2 \times 2 \) matrix-vector equation to determine the coefficients that appear in (a). DO NOT SOLVE.

2. (14 points) The equation
\[ y^{(iv)} + 16y''' + 114y'' + 400y' + 625y = 0 \] (2a)
has characteristic equation
\[ (r^2 + 8r + 25)^2 = 0 \] (2b).

Find the general solution to (2a).

3. (36 points)
(a) Given the matrix
\[ A = \begin{bmatrix} 2/3 & a_{12} & -2 \\ -1/5 & -1/3 & 3/5 \\ 1/2 & 5/6 & -3/2 \end{bmatrix}, \] (3)
for what values of \( a_{12} \) is the column space of \( A \) given by (i) a one-parameter subspace of \( \mathbb{R}^3 \), (ii) a two-parameter subspace of \( \mathbb{R}^3 \), (iii) \( \mathbb{R}^3 \)?

(b) For cases (i) and (ii), find basis vectors for the column space of \( A \).

4. (30 points)
(a) Given one solution \( y_1(x) = x \), find the general solution to the equation
\[ (x^2 - 1)y'' - 2xy' + 2y = 0, \quad x > 1. \] (4)