1. Use transformations of the graph $y = x^4$ to graph $f(x) = 2(x + 1)^4 + 1$.

**Solution:**
First shift to the left by 1 unit, then scale the y-direction by 2, then shift up by 1 unit.

2. Find the polynomial function with zeros: -1 (multiplicity 2), 1 (multiplicity 2) whose graph passes through the point $(-2, 45)$.

**Solution:** We set $f(x) = a(x + 1)^2(x - 1)^2$ and plug in $(-2, 45)$ to solve $a$

Answer: $f(x) = 5(x - 1)^2(x + 1)^2$

3. For function $f(x) = x^2(x-3)(x+4)$, (1) determine the end behavior of the graph of the function,(2) find the x- and y- intercepts of the graph of the function.(3) determine the zeros and their multiplicity. Use the information to determine whether the graph crosses or touches the x-axis at each x-intercept.(4) determine the maximum number of turning points on the graph of the function(5)Use the information from (1) to (4) to draw a complete graph of the function.
Solution:
For (1), Since the highest degree term is $x^4$, we have $f(x) \sim x^4$.
For (2), x-intercept: (0, 0), (3, 0), (−4, 0). y-intercept: (0, 0).
For (3), $x = 0$, multiplicity 2. $x = 3$ and $x = −4$ multiplicity 1. So $f(x)$ crosses the x-axis at $x = 3$ and $x = −4$, $f(x)$ touches x-axis at $x = 0$.
For (4), since $f(x)$ has degree 4, it has at most 3 turning points, and since $f(x)$ touches x-axis at $x = 0$ and must cross x-axis at $x = −4, x = 3$, it has at least 3 turning points, so it must have 3 turning points. For (5),

4. Find the vertical, horizontal and oblique asymptotes of $H(x) = \frac{x^3 − 8}{x^2 − 5x + 6}$.

Solution:
Vertical asymptotes are given by the zeros of the denominator, so set $x^2 − 5x + 6 = 0$, we get $x = 2$ and $x = 3$, so vertical asymptotes are: $x = 2, x = 3$. There is no horizontal asymptotes since the degree of the numerator is greater the degree of the denominator.
For the oblique asymptote, if $|x|$ becomes large, $H(x) \approx \frac{x^3}{x^2} = x$, so $y = x + b$. To find b, you could do long division. The whole process could also be done by long division. You would find that the oblique asymptote is $y = x + 5$.
We could also find the equation by long division.

5. Follow the steps 1 through 7 to analyze the graph of $R(x) = \frac{x(x − 1)^2}{(x + 3)^3}$. (1) find the domain of the function. (2) write $R$ in lowest terms.(3) find the intercepts of the graph and determine its multiplicity. (4) find vertical asymptotes and determine the behavior of $R$ on either side of the asymptotes.(5) find horizontal for oblique asymptote. Find possible intersections of the graph of $R$ and the asymptotes.(6) use the zeros of the numerator and denominator of $R$ to divide the x-axis into intervals. Determine where the graph of $R$ is above or below the x-axis by choosing a number in each interval and evaluation $R$ there. (7) use the results obtained in steps through 6 to graph $R$. 

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Problem:
For (1), the domain is \( x \neq -3 \).

For (2), we have \( x(x-1)^2 = (x+3)^3 - 11x^2 - 26x - 27 \), so \( R(x) = 1 - \frac{11x^2 + 26x + 27}{(x+3)^3} \).

For (3), x-intercept: \((0,0), (1,0)\). y-intercept: \((0,0)\). Since \( x = 0 \) has multiplicity 1, \( R \) crosses the x-axis at \( x = 0 \), and since \( x = 1 \) has multiplicity 2, the graph touches the x-axis at \( x = 1 \).

For (4), vertical asymptotes is: \( x = -3 \), and the function goes to positive infinity when \( x \) approaches to \(-3\) from the left side and to negative infinity when \( x \) approaches to \(-3\) from the right side.

For (5), horizontal asymptote: \( y = 1 \), to find the intersections of the graph with the asymptote, set \( R(x) = 1 \), we get \( x(x-1)^2 = (x+3)^3 \), multiplying out, we get \( x^3 - 2x^2 + x = x^3 + 9x^2 + 27x + 27 \), so we get \( 11x^2 + 26x + 27 = 0 \), and there is no real solution, so the graph doesn’t intersect the asymptote.

For (6), since \( R(-4) = 100 \), on (-\( \infty \), \(-3\)) \( R \) is positive, similarly, \( R(-1) = -\frac{1}{2} \), so on \((-3,0)\) \( R \) is negative, and \( R\left(\frac{1}{2}\right) = \frac{1}{343} \), so on \((0,1)\) \( R \) is positive, \( R(2) = \frac{2}{125} \), so on \((1, +\infty)\) \( R \) is positive.

For (7),

6. Find the rational zeros of the following polynomial and use the zeros to factor the polynomial over the real numbers.

(a) \( f(x) = x^3 + 8x^2 + 11x - 20 \).
Solution: We know that if \( \frac{p}{q} \) is a rational solution, then since our polynomial has a leading coefficient 1 and a constant term of \(-20\), we must have that \( p \) is a divisor of 20 and \( q \) is a divisor of 1, so \( q = 1 \), so we only need to check the divisors of 20, it’s easy to check that \( x = 1, x = -4 \) and \( x = -5 \) are the roots of \( f \) \( (f(1) = f(-4) = f(-5) = 0) \) and since \( f \) has degree 3, these must be all the roots of \( f \). The coefficient of \( x^3 \) is 1, so we have \( f(x) = (x - 1)(x + 4)(x + 5) \).

(b) \( f(x) = x^4 - x^3 - 6x^2 + 4x + 8 \).

Solution: By the same reasoning, if \( \frac{p}{q} \) is a rational root, then \( p \) is a divisor of 8 and \( q = 1 \). It’s easy to verify that \(-1\) and \( \pm 2 \) are the roots of \( f(x) \) and since \( f \) has degree 3, these are all the roots of \( f \). The leading coefficient of \( f \) is 1 so we have \( f(x) = (x + 2)(x + 1)(x - 2) \).