31. \( r'(t) = \sinh \frac{t}{3} i + j \)

\[ r''(t) = \frac{1}{3} \cosh \frac{t}{3} i \]

\[ T(1) = r'(1) \frac{1}{|r'(1)|} \sqrt{\sinh^2 \frac{t}{3} + 1} = \frac{\sinh \frac{1}{3} i + j}{\cosh \frac{1}{3}} = \tanh \frac{1}{3} i + \text{sech} \frac{1}{3} j \]

\[ a_T(1) = \frac{r'(1) \cdot r''(1)}{|r'(1)|} = \frac{\sinh \frac{1}{3} i + j}{\cosh \frac{1}{3}} \left( \frac{1}{3} \cos \frac{1}{3} k \right) = \frac{1}{3} \sinh \frac{1}{3} \]

\[ a_N(1) = \frac{|r'(1) \times r''(1)|}{|r'(1)|} = \left( \sinh \frac{1}{3} i + j \times \left( \frac{1}{3} \cos \frac{1}{3} k \right) \right) = \frac{1}{3} \cosh \frac{1}{3} \]

\[ a_N(1) = \frac{N(1) = r'(1) a_T(1) T(1)}{a_N(1)} = 3 \left[ \cosh \frac{1}{3} i - \frac{1}{3} \sinh \frac{1}{3} \left( \tanh \frac{1}{3} i + \text{sech} \frac{1}{3} j \right) \right] = \left( \cosh \frac{1}{3} \frac{\sinh^2 \frac{1}{3}}{\cosh \frac{1}{3}} \right) i - \tanh \frac{1}{3} j \]

\[ = \text{sech} \frac{1}{3} i - \tanh \frac{1}{3} j \]

\[ \kappa(1) = \frac{|r'(1) \times r''(1)|}{|r'(1)|^3} = \frac{\frac{1}{2} \cosh \frac{1}{3}}{\cosh^2 \frac{1}{3}} = \frac{1}{3} \text{sech}^2 \frac{1}{3} \]

\[ B(1) = T(1) \times N(1) = \left( \tanh \frac{1}{3} i + \text{sech} \frac{1}{3} j \right) \left( \text{sech} \frac{1}{3} i - \tanh \frac{1}{3} j \right) = \left( -\text{sech}^2 \frac{1}{3} - \tanh^2 \frac{1}{3} \right) k = -k \]

40. \( r'(t) = 1 + t^2 j - \frac{1}{t^2} k, t > 0 \)

\[ r''(t) = 2j + \frac{2}{t^3} k \]

\[ r'(t) \cdot r''(t) = 2t^2 - \frac{2}{t^3} = \frac{2}{t^3} (t^8 - 1) \]

\[ |r'(t)| = \sqrt{1 + t^4 + \frac{1}{t^4}} = \frac{1}{t^2} \sqrt{t^8 + t^4 + 1} \]

\[ |r'(t) \times r''(t)| = \left| \frac{4}{t^3} - \frac{2}{t^2} j + 2k \right| = \sqrt{\frac{4}{t^6} + \frac{16}{t^2} + 4t^2} = \frac{2}{t^3} \sqrt{1 + 4t^4 + t^8} \]

\[ a_T(t) = \frac{\frac{2}{t^2} (t^8 - 1)}{t^3 \sqrt{t^8 + t^4 + 1}} = \frac{2(t^8 - 1)}{t^3 \sqrt{t^8 + t^4 + 1}} \]

\[ a_N(t) = \frac{2\sqrt{1 + 4t^4 + t^8}}{t \sqrt{t^8 + t^4 + 1}} = \frac{2}{t} \sqrt{\frac{t^8 + 4t^4 + 1}{t^4 + t^8 + 1}} \]