The third exam will be given on Wednesday, April 21, during the regular lecture period (9:55 AM - 10:45 AM). The exam will cover material from Chapters 5 and 6 in the text. You may bring one sheet of paper with formulas to the exam. Be careful to copy the formulas correctly, since we will not give partial credit on the exam if you make sign errors. The following problems should provide a review of many of the topics on the exam. However, there is no guarantee that the problems on the exam will be exactly like those on this review sheet.

1. Find the exact radian measure of the angle $54^\circ$.
2. Find the exact degree measure of the angle $-\frac{5\pi}{4}$ radians.
3. Find the length of the arc in a circle of radius 9 centimeters subtended by an angle of $\frac{\pi}{12}$ radians. Find the length of the arc in a circle of diameter 4 feet subtended by an angle of 150°.
4. Find the area of a sector of a circle of radius 10 inches if the sector is determined by an angle of $\frac{\pi}{5}$ radians. Find the area of a sector of a circle of radius 12 yards if the length of the arc of the sector is 15 yards long.
5. Verify the following identities:
   (1) $\cot(\theta) + \tan(\theta) = \csc(\theta) \sec(\theta)$;
   (2) $\cos(\theta + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos(\theta) - \sin(\theta))$;
   (3) $\cot(s) - \tan(t) = \frac{\cos(s + t)}{\sin(s) \cos(t)}$;
   (4) $\sin(2x) = \frac{2 \tan(x)}{1 + \tan^2(x)}$;
   (5) $\frac{\cos^3(x) - \sin^3(x)}{\cos(x) - \sin(x)} = 1 + \sin(x) \cos(x)$;
   (6) $\sin(2x) \tan(x) = 1 + \sin^2(x) - \cos^2(x)$.
6. Find the exact value of
   (1) $\sin 75^\circ$
   (2) $\cos \frac{\pi}{12}$
   (3) $\sin 210^\circ$
   (4) $\cos \frac{-11\pi}{6}$
   (5) $\tan 15^\circ$
   (6) $\cos 165^\circ$
   (7) $\tan \frac{\pi}{3}$
7. In the following, $x$ and $t$ are angles between 0 and 2π.
   (1) If $\tan(-x) = 2$ and $\sin x < 0$, find $\sin x$ and $\cos x$. What quadrant does $x$ belong to?
   (2) If $\tan t = \frac{3}{4}$ and $\sec t < 0$, find $\sin t$ and $\cos t$. What quadrant does $t$ belong to?
   (3) Use the information from (a) and (b) to compute $\sin(x + t)$.
   (4) Use the information from (a) and (b) to compute $\cot(x + t)$.
   (5) In what quadrant is the angle $x + t$?
8. Answer the following questions:
   (1) If $\cos \theta = \frac{1}{2}$ and $\sin \theta < 0$ find the exact value of $\tan \theta$.
   (2) If $\theta$ is an angle in quadrant IV and $\cos \theta = \frac{8}{17}$, find the exact values for $\sin \theta$ and $\cot \theta$.
   (3) Given that $\tan \alpha = 2$ and $\pi \leq \alpha \leq 2\pi$, evaluate $\sin(\alpha + \frac{\pi}{3})$ exactly.
9. For each of the following, find the period, the phase shift, and when appropriate the amplitude. Also, sketch the graph of each function.
   (1) $y = \tan 4x$.
   (2) $y = -4 \sin(3x - \pi) - 3$.
   (3) $y = 2 \cos(4x - \frac{\pi}{3})$.
   (4) $y = 3 \cos(2x + \pi) + 1$.
   (5) $y = -\frac{1}{3} \cot(3x - \pi)$. 
10. Sketch the graph of each of the following:
   (1) \( y = |x| \sin(x) \);
   (2) \( y = e^x \cos(x) \);
   (3) \( y = e^{-2x} \left( \sin^2(x) - \cos^2(x) \right) \)

11. Find all solutions of each of the following equations:
   (1) \( 2 \cos x = \sqrt{3} \);
   (2) \( \tan 2x = \tan x \);
   (3) \( \sin \beta + 2 \cos^2 \beta = 1 \);
   (4) \( \sin 3u \cos u - \sin u \cos 3u = \frac{1}{2} \);
   (5) \( 2 \cos 3x = \sec 3x - \tan 3x \);
   (6) \( 2 - \cos^2 2x = 4 \sin^2 x \);
   (7) \( \sin^2 x = \frac{1}{2} \sin 2x \)

12. Suppose that \( \sin x = \frac{12}{13} \) with \( x \) in the second quadrant, and sec \( y = \frac{-5}{4} \) with \( y \) in the second quadrant. Find the exact value of each of the following:
   (1) \( \sin(x + y) \);
   (2) \( \cos(x - y) \);
   (3) \( \tan 2x \);
   (4) \( \sin \frac{y}{2} \);
   (5) \( \cos 2x \);
   (6) \( \tan \frac{y}{2} \)

13. Suppose that \( f(x) = \cos(x) \). Show that if \( h \neq 0 \)
   \[
   \frac{f(x + h) - f(x)}{h} = \cos(x) \left( \frac{\cos(h) - 1}{h} \right) - \sin(x) \left( \frac{\sin(h)}{h} \right).
   \]

14. Evaluate each of the following (you may assume that \(|x| < 1|\):
   (1) \( \cos^{-1} \left( \tan \left( \frac{\pi}{4} \right) \right) \);
   (2) \( \sin^{-1} \left( \sin \left( \frac{\pi}{4} \right) \right) \);
   (3) \( \cos^{-1} \left( \cos \left( \frac{\pi}{4} \right) \right) \);
   (4) \( \cos \left( \sin^{-1}(x) \right) \);
   (5) \( \tan \left( \cos^{-1}(x) \right) \);
   (6) \( \sin \left( 2 \cos^{-1}(\frac{\pi}{4}) \right) \);
   (7) \( \cos^{-1} \left( \cos \left( -\frac{\pi}{4} \right) \right) \).

15. Let \( f(\theta) = 3 - 4 \cos(\theta) + \cos(2\theta) \).
   (1) Show that for all angles \( \theta \) it follows that \( f(\theta) \geq 0 \);
   (2) For which angles \( \theta \) is \( f(\theta) = 0 \)?

16. In each of the following, express \( f(x) \) in terms a single cosine function. Then determine the amplitude, period, and phase shift of \( f \). Sketch the graph of \( f \).
   (1) \( f(x) = -\cos(4x) + \sqrt{3} \sin(4x) \);
   (2) \( f(x) = \cos(2x) + \sin(2x) \);
   (3) \( f(x) = \frac{1}{\sqrt{3}} \cos(\pi x) + \sin(\pi x) \).
   (4) \( f(x) = \sin x + \cos x \).
   (5) \( f(x) = \sin 2x - \sqrt{3} \cos 2x \).

17. When the top of a radio tower is observed from a distance 1000 feet from the base, the angle of elevation is 29.68°. Estimate the height of the tower.