Thus we see that the polar equation $r = \sin \theta$ describes a circle centered at $(0, \frac{1}{2})$ with radius $\frac{1}{2}$.

Because $r$ represents the distance from the origin to the point, $r$ cannot be negative. Hence for $\pi < \theta < 2\pi$, the equation $r = \sin \theta$ makes no sense because $\sin \theta$ is negative in this interval. Thus the graph of $r = \sin \theta$ contains no points corresponding to values of $\theta$ between $\pi$ and $2\pi$ (in other words, the graph contains no points below the horizontal axis).

This restriction on $\theta$ to correspond to nonnegative values of $r$ is similar to what happens when we graph the equation $y = \sqrt{x - 3}$. In graphing this equation, we do not consider values of $x$ less than 3 because the equation $y = \sqrt{x - 3}$ makes no sense when $x < 3$. Similarly, the equation $r = \sin \theta$ makes no sense when $\pi < \theta < 2\pi$.

**EXERCISES**

1. $r = \sqrt{19}, \theta = 5\pi$
2. $r = 3, \theta = 2^{1000}\pi$
3. $r = 4, \theta = \frac{\pi}{2}$
4. $r = 5, \theta = -\frac{\pi}{2}$
5. $r = 6, \theta = -\frac{\pi}{4}$
6. $r = 7, \theta = \frac{\pi}{4}$
7. $r = 8, \theta = \frac{\pi}{2}$
8. $r = 9, \theta = -\frac{\pi}{2}$
9. $r = 10, \theta = \frac{\pi}{6}$
10. $r = 11, \theta = -\frac{\pi}{6}$
11. $r = 12, \theta = \frac{11\pi}{4}$
12. $r = 13, \theta = \frac{8\pi}{4}$

In Exercises 13–28, convert the rectangular coordinates given for each point to polar coordinates $r$ and $\theta$. Use radians, and always choose the angle to be in the interval $(-\pi, \pi)$.

13. $(2, 0)$
14. $(-\sqrt{3}, 0)$
15. $(0, -\pi)$
16. $(0, 2\pi)$
17. $(3, 3)$
18. $(4, -4)$
19. $(-5, 5)$
20. $(-6, -6)$
21. $(3, 2)$
22. $(4, 7)$
23. $(3, -7)$
24. $(6, -5)$
25. $(-4, 1)$
26. $(-2, 5)$
27. $(-5, -2)$
28. $(-3, -6)$
29. Find the center and radius of the circle whose equation in polar coordinates is $r = 3 \cos \theta$.
30. Find the center and radius of the circle whose equation in polar coordinates is $r = 10 \sin \theta$.

**PROBLEMS**

31. Use the law of cosines to find a formula for the distance (in the usual rectangular coordinate plane) between the point with polar coordinates $r_1$ and $\theta_1$ and the point with polar coordinates $r_2$ and $\theta_2$.
32. Describe the set of points whose polar coordinates are equal to their rectangular coordinates.
33. What is the relationship between the point with polar coordinates $r = 5, \theta = 0.2$ and the point with polar coordinates $r = 5, \theta = -0.2$?
34. What is the relationship between the point with polar coordinates $r = 5, \theta = 0.2$ and the point with polar coordinates $r = 5, \theta = 0.2 + \pi$?
35. Explain why the polar coordinate \( \theta \) corresponding to a point with rectangular coordinates \((x, y)\) can be chosen as follows:

- If \( x > 0 \), then \( \theta = \tan^{-1} \frac{y}{x} \).
- If \( x < 0 \), then \( \theta = \tan^{-1} \frac{y}{x} + \pi \).
- If \( x = 0 \) and \( y \geq 0 \), then \( \theta = \frac{\pi}{2} \).
- If \( x = 0 \) and \( y < 0 \), then \( \theta = -\frac{\pi}{2} \).

Furthermore, explain why the formula above always leads to a choice of \( \theta \) in the interval \([-\frac{\pi}{2}, \frac{3\pi}{2}]\).

36. Give a formula for the polar coordinate \( \theta \) corresponding to a point with rectangular coordinates \((x, y)\), similar in nature to the formula in the previous problem, that always leads to a choice of \( \theta \) in the interval \([0, 2\pi)\).

\[ x = 8 \cos \frac{\pi}{4} \quad \text{and} \quad y = 8 \sin \frac{\pi}{4}. \]

Because \( \cos \frac{\pi}{4} = \frac{1}{2} \) and \( \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \), the point in question has rectangular coordinates \((4, 4\sqrt{2})\).

9. \( r = 10, \theta = \frac{\pi}{6} \)

**SOLUTION** We have

\[ x = 10 \cos \frac{\pi}{6} \quad \text{and} \quad y = 10 \sin \frac{\pi}{6}. \]

Because \( \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \) and \( \sin \frac{\pi}{6} = \frac{1}{2} \), the point in question has rectangular coordinates \((5\sqrt{3}, 5)\).

11. \( r = 12, \theta = \frac{11\pi}{4} \)

**SOLUTION** We have

\[ x = 12 \cos \frac{11\pi}{4} \quad \text{and} \quad y = 12 \sin \frac{11\pi}{4}. \]

Because \( \cos \frac{11\pi}{4} = -\frac{\sqrt{2}}{2} \) and \( \sin \frac{11\pi}{4} = -\frac{\sqrt{2}}{2} \), the point in question has rectangular coordinates \((-6\sqrt{2}, 6\sqrt{2})\).

**WORKED-OUT SOLUTIONS to Odd-numbered Exercises**

**In Exercises 1–12, convert the polar coordinates given for each point to rectangular coordinates in the xy-plane.**

1. \( r = \sqrt{19}, \theta = 5\pi \)

**SOLUTION** We have

\[ x = \sqrt{19} \cos(5\pi) \quad \text{and} \quad y = \sqrt{19} \sin(5\pi). \]

Subtracting even multiples of \( \pi \) does not change the value of cosine and sine. Because \( 5\pi - 4\pi = \pi \), we have \( \cos(5\pi) = \cos \pi = -1 \) and \( \sin(5\pi) = \sin \pi = 0 \). Thus the point in question has rectangular coordinates \((-\sqrt{19}, 0)\).

3. \( r = 4, \theta = \frac{\pi}{2} \)

**SOLUTION** We have

\[ x = 4 \cos \frac{\pi}{2} \quad \text{and} \quad y = 4 \sin \frac{\pi}{2}. \]

Because \( \cos \frac{\pi}{2} = 0 \) and \( \sin \frac{\pi}{2} = 1 \), the point in question has rectangular coordinates \((0, 4)\).

5. \( r = 6, \theta = -\frac{\pi}{4} \)

**SOLUTION** We have

\[ x = 6 \cos(-\frac{\pi}{4}) \quad \text{and} \quad y = 6 \sin(-\frac{\pi}{4}). \]

Because \( \cos(-\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \) and \( \sin(-\frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \), the point in question has rectangular coordinates \((3\sqrt{2}, -3\sqrt{2})\).

7. \( r = 8, \theta = \frac{\pi}{4} \)

**SOLUTION** We have

\[ x = 8 \cos \frac{\pi}{4} \quad \text{and} \quad y = 8 \sin \frac{\pi}{4}. \]

Because \( \cos \frac{\pi}{4} = \frac{1}{2} \) and \( \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \), the point in question has rectangular coordinates \((4, 4\sqrt{2})\).

9. \( r = 10, \theta = \frac{\pi}{6} \)

**SOLUTION** We have

\[ x = 10 \cos \frac{\pi}{6} \quad \text{and} \quad y = 10 \sin \frac{\pi}{6}. \]

Because \( \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \) and \( \sin \frac{\pi}{6} = \frac{1}{2} \), the point in question has rectangular coordinates \((5\sqrt{3}, 5)\).

11. \( r = 12, \theta = \frac{11\pi}{4} \)

**SOLUTION** We have

\[ x = 12 \cos \frac{11\pi}{4} \quad \text{and} \quad y = 12 \sin \frac{11\pi}{4}. \]

Because \( \cos \frac{11\pi}{4} = -\frac{\sqrt{2}}{2} \) and \( \sin \frac{11\pi}{4} = -\frac{\sqrt{2}}{2} \), the point in question has rectangular coordinates \((-6\sqrt{2}, 6\sqrt{2})\).

**In Exercises 13–28, convert the rectangular coordinates given for each point to polar coordinates \( r \) and \( \theta \). Use radians, and always choose the angle to be in the interval \([-\pi, \pi]\).**

13. \((2, 0)\)

**SOLUTION** The point \((2, 0)\) is on the positive \(x\)-axis, 2 units from the origin. Thus we have

\( r = 2, \theta = 0 \).

15. \((0, -\pi)\)

**SOLUTION** The point \((0, -\pi)\) is on the negative \(y\)-axis, \(\pi\) units from the origin. Thus we have

\( r = 0, \theta = -\frac{\pi}{2} \).