Name: 

You aren’t necessarily expected to finish the entire worksheet in discussion. There are a lot of problems to supplement your homework and general problem bank for studying.

**Problem 1.** Find a solution to the initial value problem:

\[
\frac{dy}{dx} = e^y x^3
\]

With initial value \( y(0) = 0 \).

**Solution 1.**

This is separable so we can write \( e^{-y} dy = x^3 dx \) and integrate both sides to find \( -e^{-y} = x^4/4 + C \). However we need \( y(0) = 0 \) so \( -e^0 = 0 + C \), thus \( C = -1 \). So the solution is \( y = -\ln(-x^4/4 + 1) \).

**Problem 2.** Find a solution to the initial value problem:

\[
\frac{dy}{dx} = y \sqrt{y^2 - 1} \cos(x)
\]

With initial value \( y(0) = 1 \).

**Solution 2.**

Once again this is separable so we write

\[
\frac{dy}{y \sqrt{y^2 - 1}} = \cos(x) dx
\]

Integrating both sides

\[
\int \frac{dy}{y \sqrt{y^2 - 1}} = \int \cos(x) dx
\]

To solve the first integral we use trig sub. Let \( y = \sec(\theta) \) then \( dy = \sec(\theta) \tan(\theta) \) substituting:

\[
\int \frac{\sec(\theta) \tan(\theta)}{\sec(\theta) \sqrt{\sec^2(\theta) - 1}} d\theta = \int d\theta = \theta = \sec^{-1}(y)
\]

We know \( \int \cos(x) dx = \sin(x) \). So this tells us that \( \sec^{-1}(y) = \sin(x) + C \). So \( y = \sec(\sin(x) + C) \). We need \( y(0) = 1 \), so \( 1 = \sec(0 + C) \), so we can take \( C = 0 \). Thus a solution to the initial value problem is \( y = \sec(\sin(x)) \).

To check that this makes sense:

\[
\frac{dy}{dx} = \sec(\sin(x)) \tan(\sin(x)) \cos(x) = \sec(\sin(x)) \sqrt{\sec^2(\sin(x)) - 1} \cos(x) = y \sqrt{y^2 - 1} \cos(x)
\]
Problem 3. Find the general solution to the differential equation
\[ \frac{dy}{dx} = x^2 + y^2 x^2 \]

Solution 3.

This is also separable, but not as obviously. We can factor our an \( x^2 \) first
\[ \frac{dy}{dx} = x^2 (1 + y^2) \]

Then we can separate and integrate
\[ \int \frac{dy}{1 + y^2} = \int x^2 dx \]

We recognize the integral on the left as \( \tan^{-1}(y) \) and we know the integral on the right is \( x^3/3 \). So
\[ \tan^{-1}(y) = \frac{x^3}{3} + C \]

This tells us that \( y = \tan(\frac{x^3}{3} + C) \) is the general solution to the differential equation.

Problem 4. Find the general solution to the differential equation (for \( x \neq 0 \)):
\[ x \frac{dy}{dx} = -y + x \]

Solution 4.

I WILL BE A SOLUTION.

Problem 5. Find the general solution to the differential equation
\[ \frac{1}{2x} \frac{dy}{dx} = y + e^{-x^2} \]

Solution 5.

I WILL BE A SOLUTION.

Problem 6. Find a solution to the initial value problem
\[ \cos(x) \frac{dy}{dx} = 1 - \sin(x)y \]

With initial value \( y(0) = 1 \).
Solution 6.

I WILL BE A SOLUTION.