(1) True or false
   (a) If \( f \) and \( g \) are differentiable, then
   \[
   \frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x).
   \]
   (b) If \( f \) is differentiable, then
   \[
   \frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}.
   \]
   (c) If \( f \) and \( g \) are differentiable, then
   \[
   \frac{d}{dx}[f(x)g(x)] = f'(x)g(x).
   \]
   (d) If \( f \) and \( g \) are continuous on \( [a, b] \), then
   \[
   \int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right) \left(\int_a^b g(x)dx\right).
   \]
   (e) If \( f \) is continuous on \([a, b]\), then
   \[
   \int_a^b x f(x)dx = x \int_a^b f(x)dx.
   \]
   (f) If \( f \) and \( g \) are continuous and \( f(x) \geq g(x) \) for \( a \leq x \leq b \), then
   \[
   \int_a^b f(x)dx \geq \int_a^b g(x)dx.
   \]

(2) Compute the area of the region between the vertical lines \( x = 0 \) and \( x = 1 \), and between the \( x \)-axis and the graph of \( y = x^3 \).

(3) Find \( \lim_{x \to 1} \frac{x^2 - 9}{x^2 + 2x - 3} \)

(4) Find the absolute maximum and absolute minimum values of \( f(x) = x^4 - 2x^2 + 3 \) on \([-2, 3]\).

(5) Find the area of the region bounded by the curves \( y = x^2 - x - 6 \) and \( y = 0 \).
(6) Find the points on the graph $y = x^2$ at which the tangent line is parallel to the line $y = 6x - 1$.

(8) Find the volume of the solid obtained by rotating the region bounded by $x^2 - y^2 = a^2$ and $x = a + h$ about the $y$-axis (where $a$ and $h$ are positive constants).

(7) Find the local extrema of $f(x) = 2x^6 - 6x^4$. Find the intervals on which the graph of $f$ is concave up or concave down, and find the $x$-coordinates of the points of inflection. Sketch the graph.

(9) Find $\lim_{v \to 4^+} \frac{4-v}{|4-v|}$

(10) Find an antiderivative for $f(x) = \frac{1}{\sqrt{1-x^2}}$

(11) Find the derivative of $xe^{\cot x}$

(12) Evaluate $\int \sin \frac{\pi x}{5} dx$
(13) Find the length of the curve

\[ y = \frac{1}{6}(x^2 + 4)^{\frac{3}{2}} \]

for \(0 \leq x \leq 3\).

(16) Find the area of the region bounded by the curves \(y = e^x - 1\), \(y = x^2 - x\), and \(x = 1\).

(14) Find \(\lim_{x \to \infty} \frac{\ln x}{\sqrt{x + \ln x}}\)

(17) If \(y = x^3 + 2x\) and \(\frac{dx}{dt} = 5\), find \(\frac{dy}{dt}\) when \(x = 2\).

(15) Find the local extrema of \(f(x) = x \ln x\). Determine where \(f\) is increasing or decreasing. Discuss the concavity, find the points of inflection, and sketch the graph of \(f\).

(18) The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm\(^2\)/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm\(^2\)?
(19) Find \( \lim_{x \to -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^2} \)

(b) The \( y \)-axis

(20) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by \( y = x^3 \) and \( y = x^2 \) about \( y = 1 \).

(c) \( y = 2 \)

(21) Differentiate \( h(\theta) = e^{\tan 2\theta} \)

(22) Evaluate \( \int_0^1 (1 - x^9)dx \)

(24) Find the absolute maximum and absolute minimum values of \( f(x) = xe^{-\frac{x^2}{8}} \), \([-1, 4]\)

(23) Find the volumes of the solids obtained by rotating the region bounded by the curves \( y = x \) and \( y = x^2 \) about the following lines:

(a) The \( x \)-axis

(b) The \( y \)-axis

(25) Find the derivative of \( f(x) = (\ln x)^{\frac{1}{5}} \)
(26) Two cars start moving from the same point. One travels south at 60\(mi/h\) and the other travels west at 25\(mi/h\). At what rate is the distance between the cars increasing two hours later?

(27) Differentiate \(y = xe^{\frac{1}{x}}\)

(28) Find the volume of the solid obtained by rotating the region bounded by \(y = 2x\) and \(y = x^2\) about the \(x\)-axis.

(29) Find the equation for the line tangent to the curve \(y = \frac{8}{4+x^2}\) at the point (2, 1)

(30) Evaluate \(\int_0^1 y(y^2 + 1)^5 \, dy\)

(31) What is the 99\(^{th}\) derivative of \(xe^x\)?

(32) Find \(\lim_{t \to 0} \frac{t^3}{\tan^3 2t}\)

(33) Evaluate \(\int_0^1 e^{\pi t} \, dt\)
(34) Find the derivative of \( f(x) = x^{\ln x} \)

(35) If \( A \) is a positive constant, then find
\[
\lim_{x \to -\infty} \frac{2x + x^2}{4x^2 - e^x}
\]

(36) Determine the area under the curve \( y = \sqrt{a^2 - x^2} \) and between the lines \( x = 0 \) and \( x = a \).

(37) Each side of a square is increasing at a rate of 8 cm/s. At what rate is the area of the square increasing when that area of the square is 16 cm\(^2\)?

(38) Compute \( \int (\sqrt{x} - \frac{3x}{\sqrt{x}} + \frac{7}{\sqrt{x^2}} - 6e^x + 1) \, dx \)

(39) Find the volume of the solid obtained by rotating the region bounded by \( x = 0 \) and \( x = 9 - y^2 \) about \( x = -1 \).

(40) Find \( \lim_{x \to \infty} \frac{2^x}{3x - 2^x} \)

(41) Evaluate \( \int \frac{x + 2}{\sqrt{x^2 + 12x}} \, dx \)
Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Find all numbers $c$ that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = e^{-2x}, [0, 3]$$

Use a substitution to evaluate

$$\int \frac{\sin 2x}{\sqrt{1+\cos 2x}} \, dx$$

Find $\lim_{x \to \infty} \frac{e^x - x^2}{e^x + e^{-x}}$.

Find the local extrema of $f(x) = 5 - 7x - 4x^2$ and the intervals on which $f$ is increasing or is decreasing, and sketch a graph of $f$.

Find the extrema and sketch the graph of $f(x) = \frac{x^2 + 2x - 8}{x + 3}$. 
(47) Evaluate $\int \frac{\sqrt{x}}{\sqrt{x}} \, dx$

(49) Differentiate $H(v) = v \arctan v$

(48) A man wishes to put a fence around a rectangular field and then subdivide this field into three smaller rectangular plots by placing two fences parallel to one of the sides. If he can afford only 1000 yards of fencing, what dimensions will give him the maximum area?

(50) The interior of a half-mile track consists of a rectangle with two semicircles at two opposite ends. Find the dimensions that will maximize the area of the rectangle.

(51) Find $\lim_{{x \to \infty}} \frac{\sqrt{x}}{\sqrt{e^x+1}}$

(52) Evaluate $\int \frac{x^3}{1+x} \, dx$

(53) Find the derivative of $f(x) = x^2 e^x$
Answers

(1) True or false
(a) True
(b) True
(c) False
(d) False
(e) False.
(f) True.

(2) 0.25
(3) $-\infty$
(4) $f(3) = 66, f(\pm 1) = 2$
(5) $\frac{125}{6}$
(6) Since the slope of $y = 6x - 1$ is 6, the slope of the tangent line must be 6. Thus the derivate $2x = 6, x = 3$. The desired point is (3, 9).

(7) $f(0) = 0$ is a maximum, $f(\pm \sqrt{2}) = -8$ are minima, concave up on $(-\infty, -\sqrt{\frac{6}{5}})$ and $(\sqrt{\frac{6}{5}}, \infty)$, concave down on $(-\sqrt{\frac{6}{5}}, -\sqrt{\frac{6}{5}})$.

(8) $\frac{1}{2} \pi (2ah + h^2)^{\frac{3}{2}}$

(9) $-1$

(10) $\arcsin x$

(11) $e^{\cot x} (1 - x \csc^2 x)$

(12) $-5 \cos \frac{\pi x}{5} + C$

(13) $\frac{15}{2}$

(14) 0

(15) Min: $f(e^{-1}) = -e^{-1}$, increasing on $[e^{-1}, \infty)$, decreasing on $(0, e^{-1})$, concave up on $(0, \infty)$, no points of inflection.

(16) $e - \frac{11}{6}$

(17) 70

(18) $-1.6 \text{cm/min}$

(19) $\frac{1}{3}$

(20) $\int_0^1 \pi [(1 - x^3)^2 - (1 - x^2)^2] \, dx$

(21) $h'(\theta) = 2 \sec^2(2\theta) e^{\tan 2\theta}$

(22) $\frac{9}{10}$

(23) (a) $\frac{2\pi}{15}$
(b) $\frac{\pi}{5}$
(c) $\frac{2\pi}{15}$

(24) $f(2) = \frac{2}{\sqrt{\pi}}, f(-1) = -\frac{1}{e^{\frac{1}{8}}}$

(25) $f'(x) = \frac{1}{5x(\ln x)^{\frac{2}{5}}}$

(26) $65 \text{mi/h}$

(27) $y' = e^{-\frac{1}{2}} (1 + \frac{1}{2})$

(28) $\frac{64}{15}$

(29) $y = -\frac{1}{2}x + 2$

(30) $\frac{21}{4}$

(31) $xe^x + 99e^x$

(32) $\frac{1}{8}$

(33) $\frac{1}{2}(e^x - 1)$

(34) $2x \ln x - 1 \ln x$

(35) $-1$

(36) $\frac{x^2}{4}$

(37) $48 \text{cm}^2/s$

(38) $\frac{2}{3} \sqrt{x^3} - \frac{3}{7} \sqrt{x^7} + 21 \sqrt{x} - 6e^x + x + C$

(39) $\frac{1656\pi}{5}$

(40) 0

(41) $\sqrt{x^2 + 4x + C}$

(42) $c = -\frac{1}{2} \ln \left( \frac{5}{6} (1 - e^{-6}) \right)$

(43) Max: $f(-\frac{7}{8}) = \frac{129}{16}$, increasing on $(-\infty, -\frac{7}{8})$, decreasing on $[-\frac{7}{8}, \infty)$

(44) $-\sqrt{1 + \cos 2x} + C$

(45) 0

(46) no extrema

(47) $2e^{\sqrt{x}} + C$

(48) 125 yards by 250 yards

(49) $H'(v) = \frac{v}{\sqrt{1 + v^2}} + \arctan v$

(50) radius is $\frac{1}{8\pi}$ mi and length is $\frac{1}{8}$ mi.

(51) 1

(52) $\frac{1}{2} \ln |1 + x^4| + C$

(53) $f'(x) = x(x + 2)e^x$