This is a Take home exam. You must work on it alone. You can use the Greenberg book as well as your notes but no other material.

1. Find the general solution to $y''' - 6y'' + 11y' - 6y = 0$. Write this equation as a 1st order system and find the general solution of the system. Sketch the phase space.

2. (a) Show that $e^{At}$, where $A$ is an n-by-n matrix and $t$ is time, solves the 1st order system $d\Phi/dt = A\Phi$ by use of the definition of the exponential of a matrix. (b) If $Ax = \lambda x$ then $e^{At}x = ?$, where $x$ is a $n$-vector and $\lambda$ a scalar.

3. Find the asymptotic behavior of the solutions of $y'' + y'/x - x^2y = 0$ for large $x$ using the WKBJ method.

4. Solve $y'' = f(x)$ with $y(0) = a$ and $y(L) = b$ using a Green’s function as well as by variation of parameters.

5. Find the general solution to $y'' - 2xy' + ny = 0$ by series solution, where $n$ is an integer.