1. (1) Derive the Green’s function for Poisson’s equation in the unbounded plane $\mathbb{R}^2$ and in 3D space $\mathbb{R}^3$.
(2) Find the Green’s function for a half plane $y > 0$ with $G = 0$ at $y = 0$ and use it to solve $\nabla^2 u = 0$, with $u(x, 0) = f(x)$.
(2) Find the Green’s function for a half plane $y > 0$ with $\partial G / \partial y = 0$ at $y = 0$. [Hints: use the method of images.]

2. Consider the Helmholtz equation $\nabla^2 u + \kappa^2 u = 0$ where $\kappa^2$ is a (positive) real number in the unbounded plane $\mathbb{R}^2$. Take a deep breath, then:
(1) Show that this equation follows from looking for eigensolutions $u(x, y, t) = \exp(\lambda t) \hat{u}(x, y)$ for both the heat equation $u_t = \nu \nabla^2 u$ and the wave equation $u_{tt} = c^2 \nabla^2 u$. Relate $\kappa$ to $\lambda$ (and $\nu > 0$ or $c^2 > 0$) for both equations and (briefly) argue on physical grounds that $\kappa^2$ should indeed be a real positive number.
(2) Show that $\kappa^2$ can be removed by re-scaling the spatial dimensions as long as we are in an unbounded domain (so there is no external length scale).
(3) Find the general solution of the rescaled equation (i.e. with $\kappa \equiv 1$) in terms of a double Fourier integral in terms of Cartesian coordinates $x$ and $y$.
(4) Transform to polar coordinates $x = r \cos \theta$, $y = r \sin \theta$, and likewise for the wavevector $k_x = k \cos \alpha$, $k_y = k \sin \alpha$, where $k_x$ and $k_y$ are the $x$ and $y$ wavenumbers, respectively. Show that the complex exponentials can be combined such that $k_x x + k_y y = kr \cos(\theta - \alpha)$.
(5) Restrict your solution to axisymmetric solutions only i.e. $u(x, y) = v(r)$. Show that $v(r)$ satisfies Bessel’s equation of order 0, therefore $v(r) = J_0(r)$. Use your Fourier approach to deduce an integral representation for $J_0(r)$.
(6) Use the method of stationary phase to deduce the leading-order asymptotic behavior of $J_0(r)$ as $r \to \infty$. 

Your write-up must be clear AND concise!