NOTE: Here are some comments I put together about what you need to study for the final exam for Lecture 1 of Math 222 this spring. I based them on (a) some information from Prof. Yang, (b) my own experience teaching Math 222, and (c) some comments from the TAs. But these have not been formally endorsed by Prof. Yang or the TAs!

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About the exam:

- Exam Room: 105 Psychology
- You are allowed to bring on 8”x10” sheet of handwritten notes, but no printed materials such as the textbook.
- You are not allowed to use a calculator.

About the mathematics:

- Techniques of integration
  - You certainly can’t get far without being able to use substitution.
  - Integration by parts: An essential tool. Be sure you know how to use it in cases where it gets used more than once, or perhaps used a couple of times and then the answer is solved for algebraically.
  - “Rational Functions”, aka Partial Fractions: There will NOT be a problem requiring this.
  - Trignometric Integrals: Integrals involving products of powers of $\sin$ and $\cos$, or products of powers of $\tan$ and $\sec$, and other integrals evaluated using trig identities.
  - “Trigonometric substitutions”: The text consistently does this using formulas, I would draw a triangle to establish the relationships. Either one should come to the same answer, except perhaps for a constant difference.
  - Numerical Integration: There will NOT be a problem requiring this.
  - Improper Integrals: Be sure to notice if the integrand “blows up” and use limits both for that situation and for the case where an endpoint is infinite. Improper integrals evaluated without explicit attention to limits will receive little if any credit.

- Conic Sections and Polar Coordinates
  - You should be able to find equations for conic sections given a geometric description, or to establish facts (e.g. foci) about and sketch a conic section from an equation.
  - You should be able to figure out the angle to rotate a conic with an equation involving an $xy$ term in order to put it in standard position. You should be able to find the equations changed by that rotation, and you should know the discriminant test.
- You should be able to find parametrizations for motion along a curve, or determine the curve from a parametrization.
- You should be able to convert between rectangular and polar coordinates.
- You should be able to recognize and to sketch the more common polar curves.
- You should be able to compute arc length, area of a plane region, and the area of a surface of revolution, for curves and regions described in polar coordinates.
- There will NOT be a problem on conic sections in polar coordinates.

- **Differential Equations**
  - You should be able to verify that a given function is (or is not) a solution to a differential equation or an initial value problem.
  - You should be able to solve separable first order differential equations and initial value problems.
  - You should be able to solve first order linear differential equations and initial value problems.
  - There will NOT be a problem requiring use of Euler’s method for numerically solving a differential equation.
  - You should be able to solve second-order, linear, constant-coefficients differential equations and initial value problems. This includes both homogeneous and nonhomogeneous cases.
  - There will be no “story problems” such are found in section 17.3.

- **Sequences and Series**
  - Infinite sequences, and series of constants:
    * Know what it means for a sequence or series to converge, and be able to apply that definition.
    * Be familiar with some special classes of series: Geometric series, harmonic and alternating harmonic series, and $p$-series. Know when they converge or diverge. For geometric series you should know how to calculate both the sum of the first $n$ terms and the sum of the series.
    * Be able to test series for convergence/divergence. You should be able to use the convergence tests described in sections §11.3-§11.5. Be sure you know what the “$n^{th}$ term test” does NOT say!
    * Be able to distinguish between absolute and conditional convergence as well as divergence for series that have terms both positive and negative.
    * Be able to use the Alternating Series Test and, going with that, the Alternating Series Estimation theorem.
  - Power Series:
    * Be able to find the radius and interval of convergence (convergence set) for a power series, including testing at end points.
    * You should be able to use geometric series, integration, and differentiation, as tools to find power series for given functions or to go from a series to a function it represents.
    * You should know common Maclaurin series such as $\cos(x)$, $\sin(x)$, and $e^x$, etc.
* You should be able to find the first few terms of the Maclaurin or Taylor series for a function, and to give a formula for the \(n^{th}\) term if there is a clear repetitive pattern among the derivatives.

* You should be able to work with the “error” resulting from using only initial terms of a series. This implies the ability to use the remainder term \(R_n(x)\) from Taylor’s theorem. In some cases you might be able to save some work by using instead the remainder term from the alternating series test.

* There will be no problem on §11.11, Fourier series.

- **Parametric Equations and Vectors**
  - You should be able to work with points in space, computing distance, finding equations for spheres, etc.
  - You need to be able to express vectors in different forms and to do arithmetic (adding, multiplying by a scalar) on vectors. Be sure to distinguish between vectors and scalars when writing your answers.
  - You should be able to find the vector from one point to another, calculate the magnitude (length) of a vector, find a unit vector, and calculate dot and cross products.
  - You should be able to find the angle between two vectors and to compute scalar and vector projections and components. This includes the ability to write a vector as a sum of parts parallel to and perpendicular to another given vector.
  - You should be able to use the dot and cross products in computing areas and volumes in space.
  - You should be able to use vectors in geometry:
    * Calculate distance from a point to a line or plane.
    * Find the line through two points or through one point and in a given direction. You should be able to restrict parametric equations to represent just part of a line, e.g. the segment from \(P\) to \(Q\).
    * Find the plane through three points, or the plane through a point and perpendicular to a given vector.
    * Find a point where a line meets a plane, or the line where two planes meet, or the angle between planes.
  - §12.6 on quadric surfaces will not be on the exam.