Math 632 Syllabus
Introduction to Stochastic Processes, Summer 2020

COURSE INFORMATION
Introduction to Stochastic Processes
MATH 632 (3 Credits)
Summer 2020

Description
632 is a survey of five important classes of stochastic processes:

- discrete-time Markov chains,
- martingales,
- Poisson processes,
- renewal processes, continuous-time Markov chains.

The material is treated at a level that does not require measure theory. Consequently technical prerequisites for this course are light: calculus, introductory probability and linear algebra are sufficient. However, the material is sophisticated, so a degree of intellectual maturity and a willingness to work hard are required.

Good knowledge of undergraduate probability at the level of UW-Madison Math 431 (or an equivalent course) is required. This means familiarity with basic probability models, random variables and their probability mass functions and distributions, expectations, joint distributions, independence, conditional probabilities and conditional expectations, the law of large numbers and the central limit theorem. Especially the multivariate topics (joint distributions, conditional expectations) are used throughout 632. If you wish to acquire a book for review, the Math 431 textbook Introduction to Probability by Anderson, Seppäläinen and Valkó is recommended.

In class we go through theory, examples to illuminate the theory, and techniques for solving problems. Homework exercises and exam problems are paper-and-pencil calculations with examples and special cases, together with short proofs.

A typical advanced math course follows a strict theorem-proof format. 632 is not of this type. Mathematical theory is discussed in a precise fashion but only some results can be rigorously proved in class. This is a consequence of time limitations and the desire to leave measure theory outside the scope of this course. Interested students can find more proofs in the literature. For a thoroughly rigorous probability course students should sign up for the graduate probability sequence Math/Stat 733-734 which requires a background in measure theory from Math 629 or 721. An undergraduate sequel to 632 in stochastic processes is Math 635 Introduction to Brownian motion and stochastic calculus. Math 635 requires undergraduate analysis Math 521 as background.

Course Designations:
Level - Advanced
Breath - Natural Sciences
L&S Credit - Counts as Liberal Arts and Science credit in L&S
Instruction mode - synchronous online

Textbooks:
- Introduction to Stochastic Processes. Lecture notes of Timo Seppäläinen and Benedek Valkó.
• *Essentials of Stochastic Processes, 3rd Ed.* by Rick Durrett.

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**Department:** MATHEMATICS  
**College:** Letters and Science  
**Canvas URL:** [https://canvas.wisc.edu](https://canvas.wisc.edu)  
**Course Webpage:** [https://sites.google.com/site/math431su2020/](https://sites.google.com/site/math431su2020/)

**Meeting Time and Location**  
MTWR 1:00PM - 2:15PM, BBCU in Canvas

**How the Credit Hours are Met**  
This class meets for four 75-minute class periods each week over the semester and carries the expectation that students will work on course learning activities (reading, writing, problem sets, studying, etc) for about 2 hours out of classroom for every class period.
Instructor: Xiaoqin Guo (xguo@math.wisc.edu)
TA: Yun Li (li724@wisc.edu) Office hours: 5:00-6:00 TuTh, in BBCU
Grader: Junyi Wei (jwei53@wisc.edu)

GRADING AND COURSE MATERIALS

Grading
Homework (25%), midterm (30%), final (45%)

- If your final scores are in the following scales the corresponding grades or better are guaranteed:
  
  \[
  A : [100, 90], AB : (90, 87), B : [87, 76), BC : [76, 74), C : [74, 62), D : [62, 50), F : [50, 0].
  \]

- **Homework:** Homework problems will be posted every Wednesday under "Assignments" in Canvas and will be due the following Wednesday at 4:00pm. Homework should be uploaded to Canvas in the form of a single PDF file. Late work is not accepted, but the lowest homework score will be dropped at the end of the semester.

  The homework is graded according to its correctness, completeness and presentation. Answers alone carry no credit. One should provide clear arguments (words like “because, since, hence, thus, therefore, by *** property/lemma/theorem, we conclude that…” are recommended to link your arguments) and steps that lead to your solution/conclusion.

  - Observe rules of academic integrity. Handing in plagiarized work, whether copied from a fellow student or off the web, is not acceptable. Plagiarism cases will lead to sanctions.
  
  - Working in groups on homework assignments is encouraged; however, everyone must write his/her own assignments.
  
  - Organize your work neatly. Use proper English. Write in complete English or mathematical sentences. Answers should be simplified as much as possible.
  
  - Recopy your problems and put them in the correct order. Do not hand in your rough draft or first attempt. Papers that are unreadable or disorganized cannot be graded.
  
  - I strongly encourage you to type up your solutions (perhaps using Latex).

- **Exams:** There is one online midterm exam and one online comprehensive final exam. Exam Schedule:
  
  Midterm Exam Jul/13/2020, Monday on Canvas
  Final Aug/06/2020, Thursday 5-8PM (CDT) on Canvas.

Course Learning Outcomes

By the end of Math 632 you should be able to:

- Recall and state the formal definitions of the mathematical objects and their properties for stochastic processes (e.g., discrete space Markov chains, Poisson processes, renewal processes, branching processes, etc.).

- Use such definitions to argue that a mathematical object does or does not have the condition of being a particular type or having a particular property (e.g., irreducibility, aperiodicity, recurrence, transience, the Markov property, etc.).

- Recall and state the standard theorems of stochastic processes. (e.g., laws of large numbers for Markov chains, existence of limiting/stationary distributions, law of large numbers for renewal processes, etc.) and recall the arguments for these theorems and the underlying logic of their proofs.
• Construct mathematical arguments related to the above definitions, properties, and theorems, including the construction of examples and counterexamples.

• Convey arguments in oral and written forms using English and appropriate mathematical terminology, notation and grammar.

• Model simple real life situations by means of discrete-space stochastic processes and calculate probabilities associated with those processes.

• Identify applications of course content in current areas of research.

**Tentative schedule:**

<table>
<thead>
<tr>
<th>Week</th>
<th>Topics</th>
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<tr>
<td>7 Jul 27</td>
<td>Invariant and reversible distributions. Limit theorem for transition probabilities and the SLLN.</td>
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<tr>
<td>8 Aug</td>
<td>Final Exam (Aug/06)</td>
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ACADEMIC POLICIES

ACADEMIC INTEGRITY
By enrolling in this course, each student assumes the responsibilities of an active participant in UW-Madison’s community of scholars in which everyone’s academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct & Community Standards for additional review. For more information, refer to https://conduct.students.wisc.edu/academic-integrity/

ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES
McBurney Disability Resource Center syllabus statement: “The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty [me] of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty [I], will work either directly with the student [you] or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student’s educational record, is confidential and protected under FERPA.”
http://mcburney.wisc.edu/facstaffother/faculty/syllabus.php

DIVERSITY & INCLUSION
Institutional statement on diversity: “Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world.” https://diversity.wisc.edu/