**Math 234 Syllabus**  
Vector (multivariable) Calculus

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**ELECTRODYNAMIC EQUATIONS**

\[
\oint_{\gamma} \vec{E} \cdot d\vec{T} = -\frac{\partial}{\partial t} \iiint_{S} \vec{B} \cdot d\vec{n} \quad \nabla \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}
\]

\[
\iiint_{\Sigma} \vec{E} \cdot d\vec{n} = \iiint_{V} \rho \, dV \quad \nabla \cdot \vec{E} = \rho
\]

\[
\iiint_{\Sigma} \vec{B} \cdot d\vec{n} = 0 \quad \nabla \cdot \vec{B} = 0
\]

\[
\oint_{\gamma} \vec{B} \cdot d\vec{T} = \mu_0 \iiint_{S} (\vec{J} + \varepsilon_0 \frac{\partial}{\partial t} \vec{E}) \cdot d\vec{n} \quad \nabla \times \vec{B} = \mu_0 (\vec{J} + \varepsilon_0 \frac{\partial}{\partial t} \vec{E})
\]

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**ELECTROSTATIC EQUATIONS**

\[
\oint_{\gamma} \vec{E} \cdot d\vec{T} = 0 \quad \nabla \times \vec{E} = 0 \iff \vec{E} = \nabla \psi
\]

\[
\iiint_{\Sigma} \vec{E} \cdot d\vec{n} = \iiint_{V} \rho \, dV \quad \nabla \cdot \vec{E} = \rho
\]

\[
\iiint_{\Sigma} \vec{B} \cdot d\vec{n} = 0 \quad \nabla \cdot \vec{B} = 0 \iff \vec{B} = \nabla \times \vec{A}
\]

\[
\oint_{\gamma} \vec{B} \cdot d\vec{T} = \mu_0 \iiint_{S} \vec{J} \cdot d\vec{n} \quad \nabla \times \vec{B} = \mu_0 \vec{J}
\]

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**EQUATIONS OF MOTION OF A CLASSICAL CHARGED PARTICLE IN AN ELECTROMAGNETIC FIELD**

\[
\ddot{\vec{r}}(t) = \frac{Q}{m} \left( \vec{E}(\vec{r}(t)) + \dot{\vec{r}}(t) \times \vec{B}(\vec{r}(t)) \right)
\]

\[Q = \text{charge of particle} \quad m = \text{mass of particle} \quad \vec{r}(t) = \text{position of particle at time } t\]
**COURSE INFORMATION**

**Vector Calculus**
MATH 234 001 (3 Credits)
Summer 2020

**Description** This course treats vectors in the plane and in three dimensional space, coordinate systems, functions of several variables (and the visualization of their graphs), differentiation of such functions and vector fields, double and triple integrals, parametrized curves and surfaces, and finally Stoke's Theorem and the Divergence Theorem of vector calculus.

**Prerequisite(s):** MATH (221 and 222 or the equivalent)

**Breadths:** N - Natural Science

**Instruction Mode:** Classroom Instruction

**Department:** MATHEMATICS

**College:** Letters and Science

**Location and Schedule for Lecture 1:** ONLINE M-R 11:45am-1:00pm

**How the Credit Hours are Met**
[Traditional Carnegie Definition] This class meets (virtually) for four 75-minute class periods each week over the summer semester and carries the expectation that students will work on course learning activities (reading, writing, problem sets, studying, etc) for about 2.5 hours out of classroom for every class meeting. The syllabus includes (see below) additional information about meeting times and expectations for student work.

**Professor:** Sean Timothy Paul stpaul@wisc.edu
Virtual Office Hours Tuesday and Thursday 3:30-4:30 Central Time

**Teaching Assistants & Section meeting times**

Gothandaraman, Asvin; sec 301 gothandarama@wisc.edu
MTWR 13:35 14:25

Sun, Yu; sec 302 ysun258@wisc.edu
MTWR 13:35 14:25

JOHNSON, Jeremy; sec 303 jjohnson99@wisc.edu
MTWR 14:35 15:25

Zhang, Ashley; sec 304 ashleyrzhang@wisc.edu
MTWR 14:35 15:25

Nicholson, Nathan; sec 305 nlnicholson@wisc.edu
MTWR 14:35 15:25

**GRADING AND COURSE MATERIALS**

**Grading**
Three midterms (33.3% each)

The grading is ‘curved’ in the sense that the average is about a BC if the grade distribution is sufficiently broad (or ‘normal’) and the average is not too high or too low. There is no set % of A’s, B’s, .... This ‘curving’ adapts to the actual cumulative scores to correct for natural variations, in exam difficulty and
grading for example, since we want you to ‘show your work’ (when applicable) and we award ‘partial credit’. We use the full grading scale from 0 to 100% with the guideline

\[ F < 30 \leq D < 50 \leq C < 70 < B < 90 < A \]

assuming the class average is about 70%.

Textbook:


Homework: Homework is not mandatory—although highly recommended—you should try and solve about 65-80 problems per week. Some of the homework is meant to be challenging and to push you to learn the material in depth. Do not expect to know what to do immediately. Try hard enough to solve the problems on your own using only the textbook and your own lecture notes. The importance of solving problems on your own cannot be overemphasized.

In fact, we have this from the dictionary:

To place excessive emphasis on: the importance of adequate preparation cannot be overemphasized.

There will be 32 Lectures (29 lectures on distinct topics plus 3 lectures towards the end devoted to solving problems), daily discussion sessions, review sessions before each exam and 3 exams.

Exams: You may consult the lectures, textbook, or your own notes. No collaboration is allowed.

- Midterm 1 July 5 10am-1pm central time; covers Lectures 1-12
- Midterm 2 July 19 10am-1pm central time; covers Lectures 13-20
- Final August 9 10am-1pm central time; covers Lectures 21-28

COURSE SCHEDULE

1. June 15: 12.1-12.3,15.7,15.8
   3d coordinate systems (Cartesian, cylindrical and spherical), distance between points
   vectors, angles and dot product.

2. June 16: 12.4-12.5
   Lines, planes and the cross product of two vectors.

3. June 17: 14.1
   Functions z = f(x,y) of two variables and their graphs.

4. June 18: 13.1,14.3,14.4
   Parametrized curves \( \vec{r}(t) \) velocity vectors \( \vec{r}'(t) \) and speed.
   Tangent plane to a graph of \( z = f(x,y) \) and partial derivatives.
5. **June 22**: 14.3, 14.4
  Partial derivatives II (computation). Linear approximation. Differentiability.
  Higher partials $f_{xy}$ and the Laplacian.

6. **June 23**: 14.5, 14.6
  Chain rule for paths and the gradient. Directional derivative.

7. **June 24**: 14.6
  Level curves and level surfaces. Tangent planes to such surfaces.
  The Gradient of $f$ is orthogonal to its level set.

8. **June 25**: 14.8
  Lagrange multipliers.

9. **June 29**: Lecture
  Gradient flow.

10. **June 30**: 16.1-16.2
    Vector fields, lines integrals and work, conservative fields.

11. **July 1**: 16.3
    Fundamental Theorem for line Integrals. The Curl of a field.
    Initial study of the rotation field.

12. **July 2**: 16.5
    Curl of a field and how it is related to conservation.
    Simply connected regions and a complete analysis of the rotation field.

13. **July 6**: 16.6
    Parametric surfaces. Spheres, Tori, and cones.

14. **July 7**: 15.3 & 15.9
    General chain rule and Jacobian matrix. Transformations.

15. **July 8**: 15.1-15.3 & 15.9
    Double Integrals (volume). Iterated integrals. Change of variables
    formula for double integrals. Switching the order in a double integral.

16. **July 9**: 15.5
    Surface integrals.

17. **July 13**: 16.7
    Flux of a field across a surface I.
18. **July 14**: 16.7
   Flux across a surface II.
   Divergence of a vector field as infinitesimal flux.

19. **July 15**: 16.8 and 16.4
   Stokes and Greens Theorem.

20. **July 16**: 16.8
    Stokes Theorem II.

21. **July 20**: 15.6
    Triple integrals (density).

22. **July 21**: Lecture
    Change of variable formula II, coordinate transformations II.

23. **July 22**: 16.9
    Triple integrals and the divergence theorem I.

24. **July 23**: 16.9
    Triple integrals and the divergence theorem II.

25. **July 27**: Maxwell’s Equations of Electromagnetism I

26. **July 28**: Lecture
    Maxwell’s Equations of Electromagnetism II

27. **July 29**: Lecture
    Maxwell’s Equations III: Plane Waves

28. **July 30**: Lecture
    Maxwell’s Equations IV: Helmholtz Decomposition

29. **August 3**: Office hour

30. **August 4**: Office hour

31. **August 5**: Office hour

32. **August 6**: Office hour
Course Learning Outcomes

- Explain what a vector in $\mathbb{R}^3$ is.
- Perform basic operations on vectors.
- Define the dot product and cross product of vectors and explain the significance of these operations.
- Provide examples of functions of several variables $f(x, y)$ and $g(x, y, z)$.
- Draw the graph of several examples of $f(x, y)$.
- Draw the level sets of several examples of $f(x, y)$.
- Explain how partial derivatives define the tangent plane to the graph of $f(x, y)$.
- State the chain rule and understand how (and when) to use it.
- Define the gradient of a function and explain its significance.
- Apply the method of Lagrange multipliers
- Define a vector field. Define “lines of force” and calculate them for simple examples.
- Define line and surface integrals. Define conservative fields and explain the connection between line integrals and conservative fields.
- State Stoke’s Theorem and Gauss’ Theorem.

ACADEMIC POLICIES

ACADEMIC INTEGRITY
By enrolling in this course, each student assumes the responsibilities of an active participant in UW-Madison’s community of scholars in which everyone’s academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct & Community Standards for additional review. For more information, refer to https://conduct.students.wisc.edu/academic-integrity/

ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES
McBurney Disability Resource Center syllabus statement: “The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty [me] of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty [I], will work either directly with the student [you] or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student’s educational record, is confidential and protected under FERPA.”
http://mcburney.wisc.edu/facstaffother/faculty/syllabus.php

DIVERSITY & INCLUSION
Institutional statement on diversity: “Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.
The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world.” https://diversity.wisc.edu/