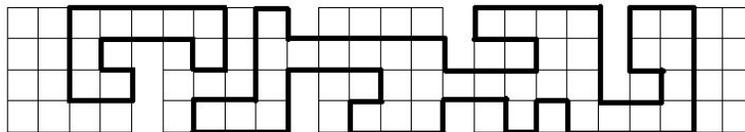


**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
 PROBLEM SET IV (2015-2016)**

January 2016

1. Alice cuts a 100 yard long piece of string into two pieces and holds onto the two cut ends. Then Becky chooses one of the two pieces of string and cuts that piece into two pieces and holds onto the two cut ends. Each player is awarded a prize proportional to the shorter of the two pieces of string she is holding. If both girls are trying to obtain the biggest prize possible, where on the string should Alice make the first cut?
2. There are 8 distinct locations in a field that a runner would like to visit. She has found a path that visits each location exactly once, such that no shorter path also does this. Show that her path does not cross itself. (Assume the locations are points in the plane and any path between them is possible.)
3. Find the smallest positive integer n so that $3n$ is a perfect cube and $5n$ is a perfect 5th power.
4. The diagram below shows a grid made up of 84 unit squares. A simple closed path on this grid follows the grid lines and forms a closed loop that never passes through any point more than once. Shown with bold are two simple closed paths, one with length 20 and one with length 66. Find the greatest possible length of a simple closed path on this grid.



5. Let $A_1A_2A_3A_4A_5A_6A_7$ be a regular heptagon. The diagonals $\overline{A_1A_3}$ and $\overline{A_2A_7}$ intersect at point B , $\overline{A_3A_5}$ and $\overline{A_4A_6}$ intersect at point C , and $\overline{A_1A_4}$ and $\overline{A_3A_7}$ intersect at point D . Show that B , C and D are on the same line. (You should start by sketching a picture.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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